Heinemann VCE Zone Mathematical Methods 1 & 2 CAS

Study Guide

1. Linear functions

Summary

Equations

- When solving linear equations you must collect all the unknowns on one side first and then undo or reverse the operations.
- If fractions are present you must find the lowest common denominator before collecting terms.
- Simultaneous equations can be solved in five different ways.
  1. By substitution: substituting one equation into the other equation and then solving.
  2. By elimination: eliminating one of the unknowns by adding or subtracting the two equations. (The coefficients of the unknown that is being eliminated must be the same.)
  3. Graphically: sketch both lines on the same set of axes and then find the point of intersection.
  4. Using matrices, where $X = A^{-1} \times B$.
  5. Using the solve function on your CAS.

Coordinate geometry

- The gradient of the straight line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.
- The angle of inclination of a line is found by $\tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$ where θ is the angle (measured in an anticlockwise direction) that the line makes with the positive direction of the x-axis ($0° \leqslant \theta < 180°$).
- The general equation of a straight line is $y = mx + c$ where m is the gradient of the line and c is the y-intercept.
- The equation of a line with a gradient of m and passing through the point (x_1, y_1) is given by $y - y_1 = m(x - x_1)$.
- If the line passes through the points (x_1, y_1) and (x_2, y_2) then the equation of the line is found by $y - y_1 = m(x - x_1)$ where $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.
- The distance between two points (x_1, y_1) and (x_2, y_2) is Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- The midpoint of a line joining (x_1, y_1) and (x_2, y_2) is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.
- If two lines are parallel then $m_1 = m_2$.
- If two lines are perpendicular to one another then $m_1 m_2 = -1$, i.e. $m_1 = -\dfrac{1}{m_2}$.

Set notation

- { } contains the elements of a set
- $\in$ means 'is an element of'
- $\cap$ = intersection (common to both sets)
- $\cup$ = union (the elements of both sets)
- $\varnothing$ = null set (empty set)
- $[a, b] = a \leqslant x \leqslant b$
- $(a, b) = a < x < b$
- $[a, b) = a \leqslant x < b$
- $(a, b] = a < x \leqslant b$
- R is the set of real numbers.
- Q is the set of rational numbers.
- J is the set of integers.
- N is the set of natural numbers.
- R^+ is the set of positive real numbers, R^- is the set of negative real numbers.
- $R^+ \cup \{0\}$ is the set of positive real numbers and zero.
- A relation is a set of ordered pairs where (a, b) is an ordered pair.
- A function is a relation in which no two ordered pairs have the same first element, i.e. any vertical line will pass through only one point on the graph.
- The domain of a function is the set of all the first elements of the ordered pairs. It is normally stated in the definition of the function. If not stated it is represented by the maximal domain possible (all the possible x values for which the function exists).
- The range of the function is the set of all the second elements of the ordered pairs (normally the y values).

Transformations using matrices

- When the point (x, y) is translated a units right and b units up it finishes at some point (x', y'). The matrix representation for this is $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} x + a \\ y + b \end{bmatrix}$
- There are three types of reflection to consider. In each case the point (x, y) is mapped onto the point (x', y').
  – reflection in the x-axis: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$
  – reflection in the y-axis: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$
  – reflection in the line $y = x$: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix}$

- There are two types of dilation to consider. In each case the point (x, y) is mapped onto the point (x', y').
  - dilation by a factor k from the x-axis:
    $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ ky \end{bmatrix}$$
  - dilation by a factor k from the y-axis:
    $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} kx \\ y \end{bmatrix}$$

Frequently asked questions

How do I know if I am to use open or closed dots when drawing my graphs?
If the point is included then the dot is closed (filled in). If the point is not included then the dot is open (not filled in).

How do I set up an equation or rule when dealing with linear modelling?
Look for the fixed cost (the quantity that will not change). This is c in the equation $y = mx + c$. The quantity that changes depending on use will be m.

Is it necessary to be able to solve, by hand, simultaneous equations using matrices?
Yes, you must develop the whole suite of manual skills as well as CAS skills. Remember that you will have to undertake technology-free examinations.

Study notes

- To find the range of a linear function with a restricted domain, substitute the value of x into the function to find the starting point. The range will then depend on the slope of the function and the restriction.
  For example: If $x > a$, then the range will be $y > f(a)$ if $f(x)$ has a positive gradient and $y < f(a)$ if $f(x)$ has a negative gradient.
  If $x < a$, then the range will be $y < f(a)$ if $f(x)$ has a positive gradient and $y > f(a)$ if $f(x)$ has a negative gradient.
- If x is included in the domain, then y is included in the range.
- If sketching graphs by hand, select the most appropriate method.
  For example, if the function is in the form $y = mx + c$, use the gradient and y-intercept method.
  Express m in the form $\frac{\text{rise}}{\text{run}}$.
  If the function is in the form $ax + by + c = 0$, or $ax + by = c$, use the x- and y-intercept method.
- For $ax + by + c = 0$, the x-intercept is $(-\frac{c}{a}, 0)$ and the y-intercept is $(0, -\frac{c}{b})$.

Cumulative Practice Examination 1

30 mins   Chapter 1   Total marks: 17

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

1 Solve the following equation.
$$\frac{3(2x-5)}{7} - 3 = \frac{4(5-2x)}{3} + \frac{1}{2}$$ (3 marks)

2 Find the equation of the line that:
(a) passes through the point (-3, 1) and is parallel to the line $4y - 5x + 1 = 0$
(b) passes through the midpoint of (-2, 4) and (6, 10) with a gradient of $-\frac{1}{3}$. (2 + 2 = 4 marks)

3 Two containers contain a total of 800 litres between them. The liquid flows from one container to another. One container holds 200 litres less than three times the contents of the other container.
(a) Set up a set of simultaneous equations to model this situation.
(b) Find the amount in each container.
(2 + 2 = 4 marks)

4 Find the value(s) of y so that the distance between the points $(5, y)$ and $(8, -1)$ is 5 units.
(2 marks)

5 A line passes through the midpoint of the interval AB where A is the point (-3, 11) and B is the point (7, -7) such that it makes an angle of 135° with the positive direction of the x-axis.
(a) Find the midpoint of AB.
(b) Find the gradient of the line.
(c) Find the equation of the line.
(1 + 1 + 2 = 4 marks)

Cumulative Practice Examination 2

45 mins Chapter 1 Total marks: 26

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

Multiple choice

1 The gradient and the y-intercept of the line with equation $3x - 2y + 8 = 0$ are:

A $m = 3, c = 8$ **B** $m = -\frac{3}{2}, c = -4$

C $m = \frac{3}{2}, c = 4$ **D** $m = \frac{3}{2}, c = -8$

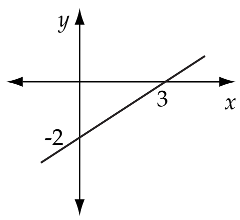
E $m = \frac{3}{2}, c = \frac{8}{3}$

2 The range of the function $f: x \leqslant 1 \to R$ where $f(x) = 5 - 2x$ is:

A $(-\infty, 3)$ **B** $[3, \infty)$ **C** $(3, \infty)$

D $(-\infty, 3]$ **E** R

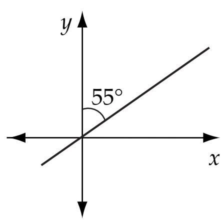
3 The equation of the line shown is:



A $y = \frac{3}{2}x - 2$ **B** $y = -\frac{3}{2}x + 3$ **C** $y = -\frac{2}{3}x - 2$

D $y = \frac{2}{3}x + 3$ **E** $y = \frac{2}{3}x - 2$

4 The gradient of the given line is:



A 55° **B** tan 55° **C** 35°

D tan 35° **E** -tan 55°

5 The gradient of a line that is parallel to $5x - 3y = 11$ is:

A $\frac{3}{5}$ **B** 5 **C** -5 **D** $-\frac{3}{5}$ **E** $\frac{5}{3}$

6 The equation of the line perpendicular to the line $y + 3x - 5 = 0$ that passes through the point (2, 3) is:

A $y = -3x + 9$ **B** $3y - x - 7 = 0$

C $3y + x - 11 = 0$ **D** $y = 3x - 3$

E $y = \frac{1}{3}x + 3$

7 The solution to $3(x - 2) = 4 - 2x$ is:

A $-\frac{2}{5}$ **B** 2 **C** $\frac{6}{5}$ **D** 10 **E** 5

8 The equation of a line with a gradient of -2 that passes through (-1, 3) is:

A $y = -2x + 5$ **B** $y = -2x - 5$ **C** $y = -2x - 1$

D $y = -2x + 1$ **E** $y = 3x - 2$

9 The solution to the set of equations $2x - 3y = 1$ and $y = 3x + 2$ is:

A (-2, -4) **B** (2, 1) **C** (0, 2)

D (-1, -1) **E** (2, 8)

10 The midpoint of the line joining (-2, 3) and (3, -5) is found by evaluating:

A $\left(\frac{-2-3}{2}, \frac{3-5}{2}\right)$ **B** $\left(\frac{-2+5}{2}, \frac{3-3}{2}\right)$

C $\left(\frac{-2+3}{2}, \frac{3-5}{2}\right)$ **D** $\left(\frac{-2-3}{2}, \frac{3+5}{2}\right)$

E $\left(\frac{-5-3}{2}, \frac{3+2}{2}\right)$

11 The distance between (-1, 3) and (4, 7) is found by evaluating:

A $(7-3)^2 + (4+1)^2$ **B** $\sqrt{(7+1)^2 + (4-3)^2}$

C $\sqrt{(-1-4)^2 + (3-7)^2}$ **D** $\sqrt{(7-3)^2 + (4+1)^2}$

E $\sqrt{(7-4)^2 + (3+1)^2}$

12 The equation of the line passing through the points (3, -5) and (6, 5) is:

A $y = \frac{10}{3}x - 5$ **B** $3y - 10x - 15 = 0$

C $3y - 10x + 45 = 0$ **D** $10y - 3x + 59 = 0$

E $y = 10x - 15$

Extended answer

1 A car travelling at a constant speed of 98 km/h on a freeway passes a sign that states that the next town is 80 km away.

(a) Find a model to show the distance from the town, t minutes after passing the sign. (2 marks)

(b) A second car passes the sign 10 minutes after the first car. If it was travelling at 105 km/h find a model for its distance from the town. (3 marks)

(c) What time (to the nearest minute) did the second car pass through the town if the first car passed the sign at 12 noon? (2 marks)

(d) How many minutes earlier did the first car pass through the town? (2 marks)

(e) Assuming that both cars travelled at their constant speeds, find where the second car caught up with the first car. (2 marks)

(f) Sketch the graph of the distance travelled by both cars over the first three hours and hence find their distance apart at the end of this time. (3 marks)

2. Quadratic functions

Summary

Polynomials

- A polynomial is an algebraic expression that contains non-negative whole number powers of a variable.
- The $f(x)$ notation can describe any algebraic expression. $f(3)$ would require the substitution of 3 for x.
- Two polynomials are identically equal (symbol $\equiv$) if all coefficients are the same.

Expansion

- Expanding: $(x + a)(x + b) = x^2 + bx + ax + ab$
- Expanding perfect squares: $(x + a)^2 = x^2 + 2ax + a^2$

Factorisation

- Perfect squares: $a^2 + 2ab + b^2 = (a + b)^2$
  $a^2 - 2ab + b^2 = (a - b)^2$
- Difference of two squares: $a^2 - b^2 = (a - b)(a + b)$

Solving equations

- The null factor law: if $a \times b = 0$ then $a = 0$ or $b = 0$, or both $a = 0$ and $b = 0$
  So if $(x + b)(x - c) = 0$ then $x = -b$ or $x = c$
- The quadratic formula is used to solve quadratic equations. If $ax^2 + bx + c = 0$ then:
  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- The discriminant $\Delta = b^2 - 4ac$ indicates the type of solution.
  $\Delta = 0$: one solution
  $\Delta > 0$: two solutions
  $\Delta < 0$: no solutions
- If $y = ax^2 + bx + c$ is graphed then the value of the discriminant shows how many x-intercepts the graph will have.

Parabolas

- The graph of a quadratic in the form $y = a(x - h)^2 + k$ is a parabola with turning point of the parabola at (h, k)
  - a represents the dilation factor parallel to the y-axis If $a < 0$ the graph is inverted with a maximum turning point
  - h represents the translation horizontally (h positive moves right, h negative moves left)
  - k represents the translation vertically (k positive moves up, k negative moves down)
- To obtain a quadratic in this form, complete the square.
- A quadratic in the form $y = a(x - b)(x - c)$ has x-intercepts at b and c.
- A quadratic in the form $y = ax^2 + bx + c$ has a y-intercept of c.

Frequently asked questions

To find the turning point do I always have to complete the square?
No. If the x-intercepts are known then the x-coordinate of the turning point will be halfway between these, and the y-coordinate can be found by substituting into the equation.

When solving a quadratic equation, will I get the right answer if I use the quadratic formula?
There are often simpler ways of finding the solutions for x than using the quadratic formula, but it will always give the correct answer.

Study notes

- When factorising, always look for common factors first, then difference of two squares or perfect squares. Then use other methods.
- Know how to factorise, expand and solve equations using CAS.
- Remember that the null factor law only works if the equation equals 0.
- Make sure you understand all the different forms of a quadratic equation and how to get an equation into each form. Relate the factorising, completing the square and the solving of quadratic equations to the graphs.

RETURN TO MAIN MENU

Cumulative Practice Examination 1

30 mins | Chapters 1 & 2 | Total marks: 17

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

1 Solve the following simultaneous equations.

(a) $4x - 3y = 4$
$2y - 5x = 2$

(b) $5x - 3y = 13$
$y = 2x - 3$

(3 + 2 = 5 marks)

2 Determine the exact values of a such that the function $y = x^2 + 3ax + 5$ intersects the x-axis at only one point. (2 marks)

3 The graph $y = mx^2 + nx + p$ is the graph of $y = x^2$ dilated by a factor of a half parallel to the y-axis, translated 4 left and 2 down. Find the values of m, n and p. (3 marks)

4 For $y = 2x^2 + 3x + 4$, find:

(a) the coordinates of the y-intercept
(b) the coordinates of the x-intercepts
(c) the equation in turning point form by completing the square.
(d) Draw the graph of the parabola showing key features. (1 + 1 + 2 + 1 = 5 marks)

5 If $m^2 + 4m + 3 = 0$, then:

(a) factorise the equation
(b) find the solution(s) for m (1 + 1 = 2 marks)

Cumulative Practice Examination 2

45 mins | Chapters 1 & 2 | Total marks: 26

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

Multiple choice

1 The gradient and the y-intercept of the line with equation $5x + 3y + 10 = 0$ are:

A $m = 5, c = 10$ **B** $m = -\frac{5}{3}, c = -\frac{10}{3}$
C $m = \frac{3}{5}, c = \frac{10}{3}$ **D** $m = \frac{5}{3}, c = -10$
E $m = -\frac{5}{3}, c = \frac{10}{3}$

2 The range of the function f: [-3, 5) $\rightarrow R$ where $f(x) = 4x - 3$ is:

A R **B** [-3, 5) **C** [-15, 17]
D [-9, 17) **E** [-15, 17)

3 The gradient of the line perpendicular to the line $\frac{5x - 7y}{2} = 8$ is:

A $-\frac{5}{2}$ **B** $\frac{5}{7}$ **C** $-\frac{2}{5}$ **D** $-\frac{7}{5}$ **E** $\frac{14}{5}$

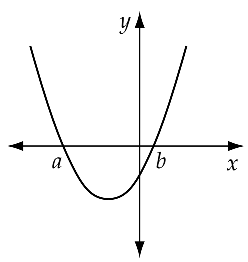
4 The equation of the line that passes through (-1, 2) and (3, 4) is:

A $2y - x - 3 = 0$ **B** $2y - x - 5 = 0$ **C** $y = x + 3$
D $3y - x - 7 = 0$ **E** $y = \frac{1}{2}x - 2$

5 The solutions to the equation $2(x + a)^2 - b = 0$ are -5 and -1. The values of a and b are:

A $a = 3, b = 8$ **B** $a = 3, b = -4$ **C** $a = 3, b = 4$
D $a = 3, b = -8$ **E** $a = -3, b = -8$

6 The equation of the graph shown could be:



A $y = (x + a)(x - b)$ **B** $y = 3(x + a)(x - b)$
C $y = 2(x - a)(x - b)$ **D** $y = (x - a)(x + b)$
E $y = (x + a)(x + b)$

7 If the graph $y = x^2$ is translated left 3 and dilated by a factor of 2, the equation of the resultant graph would be:

A $y = 2x^2 + 3$ **B** $y = 2(x + 3)^2$
C $y = (x + 3)^2 + 2$ **D** $y = 2(x - 3)^2$
E $y = 2x^2 - 3$

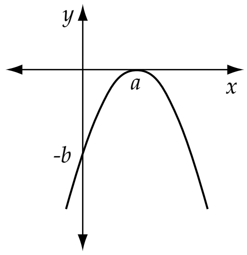
8 If the curve $y = x^2$ is dilated by a factor of $\frac{1}{2}$ and translated 3 left, the equation of the resultant curve would be:

A $y = \frac{1}{2}x^2 + 3$ **B** $y = 2(x - 3)^2$ **C** $y = \frac{1}{2}x^2 - 3$
D $y = \frac{1}{2}(x - 3)^2$ **E** $y = \frac{1}{2}(x + 3)^2$

9 The range of the function $y = -2x^2 + 3x - 5$ is:

A $(-\infty, -3.875]$ **B** $[-3.875, 0.75]$ **C** $(-\infty, 0.75]$
D $[-3.875, \infty)$ **E** $(-\infty, -3.875)$

10 The graph shown could have equation:

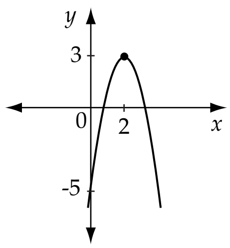


A $y = (x - a)^2 + b$ **B** $y = (x + a)^2 + b$
C $y = x^2 + 2a + b$ **D** $a^2y - bx^2 - \frac{2b}{a} + 1 = 0$
E $y = \frac{-b}{a^2}(x - a)^2$

11 $(x + 2)^2 + a(x + 2) + b$ factorises to $(x + 1)(x + 6)$. The values of a and b are:

A $a = 7, b = 6$ **B** $a = 3, b = 4$
C $a = 5, b = -8$ **D** $a = 7, b = -6$
E $a = 3, b = -4$

12 The equation for the graph shown could be:



A $y = -2x^2 + 8x - 5$ **B** $y = -2x^2 + 8x + 11$
C $y = -x^2 - 4x + 7$ **D** $y = -x^2 - 8x + 5$
E $y = -2x^2 + 8x + 5$

Extended answer

1 The profitability of a business (in dollars per day) is modelled by the equation $p(t) = -0.5t^2 + 5t + 180$, where t represents the number of additional staff employed.

(a) Determine the profit being made when no additional staff are required. (1 mark)

(b) Determine the profit being made when two new staff are employed. (1 mark)

(c) What happens if 30 new staff are employed? (1 mark)

(d) Find when the profit will equal 0. (2 marks)

(e) Write the function $p(t)$ in turning point form. (2 marks)

(f) Hence determine the number of additional staff required for maximum profitability. (1 mark)

(g) Draw the graph of the function, clearly showing the appropriate domain. (3 marks)

(h) The manager is happy as long as the profit is at least \$150 per day. Determine the range of values that the number of additional staff employed can be between. (Assume that part-time workers are being used, so your answer should be given correct to two decimal places.) (3 marks)

3. Higher order polynomial functions

Summary

Transposing

- When transposing a formula to change the subject, we always perform the same operation to the right-hand side as we do to the left-hand side.

Perfect cubes

- The perfect cubes are $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ and $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

Remainder theorem and factor theorem

- The remainder theorem tells us that $P(-a)$ is the remainder when $P(x)$ is divided by $(x + a)$.
- If the remainder is zero, then $(x + a)$ is a linear factor of $P(x)$. This is the *factor theorem*.

Factorising cubics

- To factorise a cubic:
  1. Call it $P(x)$.
  2. Use the factor theorem to find the first linear factor.
  3. Use long division to find the quadratic factor.
  4. Factorise the quadratic if possible.

Sum and difference of cubes

- Special cubics are the sum of cubes $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ and the difference of cubes $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

Solving cubic equations

- When solving cubic equations of the form $ax^3 + bx^2 + cx + d = 0$:
  1. Use the factor theorem to find the first factor.
  2. Use long division or inspection to find the quadratic factor.
  3. Factorise the quadratic (if possible).
  4. Use the Null Factor Law to solve for x.

  If you cannot find the first factor use your CAS to find the root(s).
  For $a(x-h)^3 + k = 0$ use backtracking or the balance method to solve for x.

Sketching graphs of polynomial equations

- When sketching graphs of cubic equations of the form $y = ax^3 + bx^2 + cx + d$:
  1. Determine the basic shape and sign of the graph.

     If $a > 0$ then  If $a < 0$ then 
  2. Find the y-intercept (let $x = 0$).
  3. Find the x-intercept(s). (Let $y = 0$ and use factorisation and the null factor law.)
  4. Locate the turning point(s), if necessary by using your CAS.
  5. Sketch the graph.

  Note: If the cubic is factorised into the form $(x-a)^2(x-b)$ then the graph will have a turning point at the x-intercept of $x = a$.
- When sketching graphs of quartic equations of the form $y = ax^4 + bx^3 + cx^2 + dx + e$:
  1. Determine the basic shape and sign of the graph.

     If $a > 0$ then  If $a < 0$ then 
  2. Find the y-intercept (let $x = 0$).
  3. Find the x-intercept(s). (Let $y = 0$ and use factorisation and the null factor law.)
  4. Locate the turning point(s), if necessary by using your CAS.
  5. Sketch the graph.

  Note: If the quartic has a factor repeated three times, i.e. it is of the form $(x-a)^3(x-b)$, there will be a point of inflection on the x-axis at a.
- To sketch graphs of the type $y = a(x-h)^n + k$, $n = 3$ or 4, follow these steps.
  1. Note the sign of the function to determine the shape of the graph.

     For cubic graphs  if positive or  if negative.

     For quartic graphs  if positive or  if negative.
  2. Calculate the new position of the point (0, 0); it will now be at (h, k).
  3. Calculate the x- and y-intercepts (if they exist).
  4. Sketch the graph.
- A cubic function with no restrictions will have domain of R and range of R.
- A quartic function with no restrictions will have domain R but will always have a restricted range.

RETURN TO MAIN MENU

Frequently asked questions

Can I use my CAS to completely factorise cubics? Why do I need to learn how to do it algebraically?
The CAS can be used to find factors for cubic functions. However, the technology-free examination will require you to possess by-hand skills in this area.

How can I tell what sort of equation best fits a set of points?
Use your CAS to plot the points to look at the basic shape. You can also use finite difference tables.

Study notes

- The general solution to an equation of the form $0 = a(x - h)^3 + k$ is $x = \sqrt[3]{-\frac{k}{a}} + h$.
- Practise the inspection method for finding the quadratic factor when you have to factorise a cubic function. It is much quicker than using long division.
- Remember DRT (dilations, reflections, translations): this is the order in which transformations should be performed.
- When determining the transformations that a function has undergone from its base function, always make sure that it is in standard form.
- Use your CAS to assist with graphs but make sure that you know the basic shapes without it.

Cumulative Practice Examination 1

30 mins   Chapters 1–3   Total marks: 17

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

1 Find the turning point of the graph of $y = 3x^2 - 12x + 17$. (2 marks)

2 State the domain and range of
(a) $y = 15 - 3(7 - x)^3, x \leqslant -2$
(b) $y = -2(x - 2)^4 + 3, x > -2$ (2 + 2 = 4 marks)

3 Solve $3x^3 - 20x^2 + 23x + 10 = 0$, showing all working. (2 marks)

4 **(a)** Factorise $2a^3 - 16b^3$.
(b) Hence factorise $2(x - 1)^3 - 16(x + 2)^3$. (2 + 2 = 4 marks)

5 Factorise the following expressions.
(a) $2x^4 - 12x^3 + 6x^2 + 52x - 48$ into four linear expressions
(b) $81x^4 - 450x^2 + 625$ as the product of two quadratic factors (3 + 2 = 5 marks)

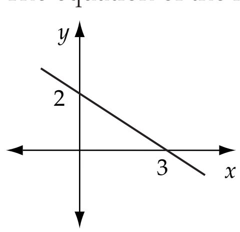
Cumulative Practice Examination 2

60 mins   Chapters 1–3   Total marks: 37

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

Multiple choice

1 The equation of the line shown is:



A $y = \frac{2}{3}x + 2$ **B** $y = -\frac{3}{2}x + 3$
C $y = -\frac{2}{3}x + 2$ **D** $y = -\frac{2}{3}x + 3$
E $y = \frac{3}{2}x + 2$

2 The domain and range of the function $f: [-2, 3) \to R$ where $f(x) = -2x + 3$ are:
A $[-2, 3), R$ **B** $[-2, 3), (-3, 7)$
C $[-2, 3), (-3, 7]$ **D** $[-2, 3), [-3, 7)$
E R, R

3 A line with the equation $y - 2 = 3(x + 1)$ has:
A a gradient of 3 and a y-intercept of 1
B a gradient of -2 and a y-intercept of 5
C a gradient of -3 and a y-intercept of 2
D a gradient of 3 and a y-intercept of 5
E a gradient of 1 and a y-intercept of -5

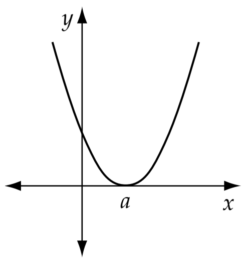
RETURN TO MAIN MENU

4 The coordinates of the turning point of $y = (x + a)(x + b)$ are (-1, 9). The values of a and b could be:

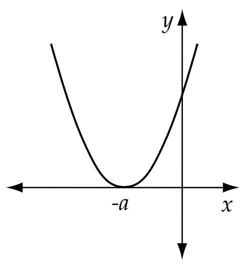
A $a = -2, b = -4$ **B** $a = 4, b = -2$
C $a = -1, b = -9$ **D** $a = -4, b = -2$
E $a = 2, b = -4$

5 Which of the following could be the graph of $y = (x - a)^2$?

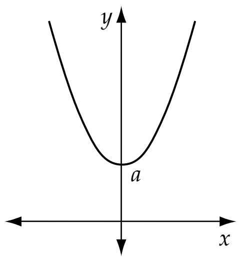
A



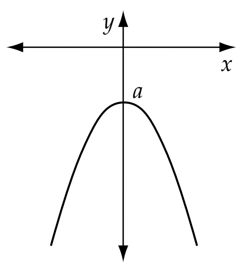
B



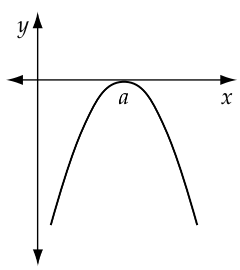
C



D



E



6 For the graph of $y = (x - a)^3(x - b)$, where a and $b > 0$, which of the following statements is false?

A There are two x-intercepts: $x = a$ and $x = b$.
B The y-intercept occurs at ab.
C There is a point of inflection at $x = a$.
D As $x \to \infty, y \to \infty$.
E The graph has a minimum turning point in the fourth quadrant.

7 The expansion of $\left(\frac{5w}{2} - \frac{1}{w^3}\right)^3$ is:

A $\frac{25w^2}{4} - \frac{5}{w^2} + \frac{1}{w^6}$

B $\frac{125w^3}{8} - \frac{1}{w^9}$

C $\frac{125w^3}{8} + \frac{75}{4w} + \frac{15}{2w^5} + \frac{1}{w^9}$

D $\frac{125w^3}{8} - \frac{75}{2w} + \frac{15}{2w^6} - \frac{1}{w^9}$

E $\frac{125w^3}{8} - \frac{75}{4w} + \frac{15}{2w^5} - \frac{1}{w^9}$

8 If $(ax - b)$ and $2x^2 + 3x - 2$ are factors of $6x^3 + 5x^2 - 12x + 4$ then a and b are:

A $a = 3, b = 2$ **B** $a = -3, b = -2$
C $a = -3, b = 2$ **D** $a = 2, b = -3$
E $a = 3, b = -2$

9 The range of the function $y = 2(x + 3)^3 - 1, x \geqslant 2$ is:

A $y > 729$ **B** $[2, \infty)$ **C** $[249, \infty)$
D $(-\infty, 249]$ **E** $y \geqslant 53$

10 Given that $w = \frac{1}{4}mp^2$ then p is equal to:

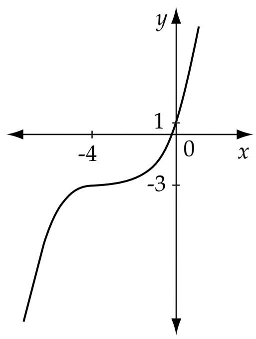
A $\frac{4w}{m}$ **B** $\pm\frac{\sqrt{4w}}{m}$ **C** $\pm\frac{1}{2}\sqrt{\frac{w}{m}}$

D $\pm 2\sqrt{\frac{w}{m}}$ **E** $\pm\sqrt{\frac{m}{4w}}$

11 If $x - 3$ is a factor of $x^3 + ax^2 - x - 6$, then a is equal to:

A 2 **B** -2 **C** 1 **D** -1 **E** -3

12 A possible equation for the following graph is:

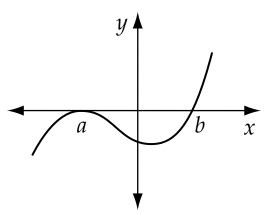


A $y = (x - 4)^3 + 3$ **B** $y = \frac{1}{3}[(x + 4)^3 - 3]$

C $y = (x + 4)^3 - 3$ **D** $y = \frac{1}{16}(x + 4)^3 - 3$

E $y = \frac{1}{9}(x + 3)^3 - 4$

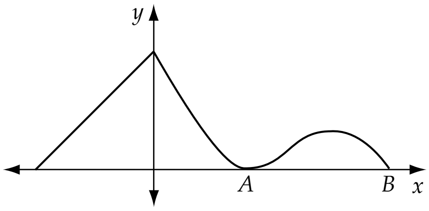
13 If $a < 0$ and $b > 0$, a possible equation for the graph shown is:



A $y = (x + a)^2(x - b)$ **B** $y = (x + a)(x - b)^2$
C $y = -(x - a)^2(x - b)$ **D** $y = (a - x)^2(x - b)$
E $y = (x + a)^2(b - x)$

Extended answer

1 A proposed amusement ride consists of a canoe being pulled up a steep incline and then released down a curved path as shown. Distances are in metres.



The curved part is modelled by the equation $y = -0.02(x^3 - 38x^2 + 441x - 1620)$, $x \in [0, B]$

(a) Find the value of B. (2 marks)

(b) Find the value of A. (1 mark)

(c) How high is the point at which the curved section begins? (1 mark)

(d) Find the height of the ride 3 m to the right of where the curved section begins. (1 mark)

(e) Find the point at which the curved section is 3 m off the ground. (2 marks)

(f) Find the equation of the inclined path if it has a gradient of $\frac{4}{5}$. (2 marks)

(g) Find the horizontal distance covered by the inclined path. (1 mark)

2 Jason is building a dome-shaped display cabinet with its edge modelled by the equation $a(x) = -0.5x^2 + 90$ and all distances in cm. Assume that the base of the cabinet is the x-axis.

(a) Find the length of the base of the cabinet to the nearest cm. (2 marks)

(b) If shelves are to be built at 20 cm intervals (from the base), find the coordinates of each end of the four shelves. (4 marks)

(c) Find the length of each of the shelves. (2 marks)

(d) Jason likes the look of the finished cabinet and decides to build another that is only 40 cm high and has a base length of 30 cm. Find the equation that could model the edge of this cabinet. (2 marks)

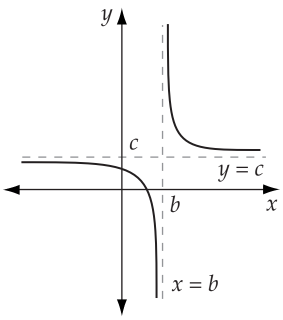
(e) If shelves are built at intervals of 15 cm from the base, state the coordinates of each end of two shelves and find the length of wood required to build them. (4 marks)

4. Advanced functions and relations

Summary

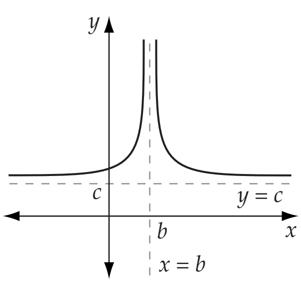
Hyperbola

- The general equation of a hyperbola is
  $y = \dfrac{a}{x-b} + c,\ x \neq b$
  where: a is the dilation factor from the x-axis or parallel to the y-axis
  b is the horizontal translation
  c is the vertical translation
  a negative value of a means that the standard graph has been reflected in the x-axis.
- When graphing hyperbolas important points to show are asymptotes and intercepts.
- Asymptotes occur at $x = b$ and $y = c$ if the equation is in the general form.



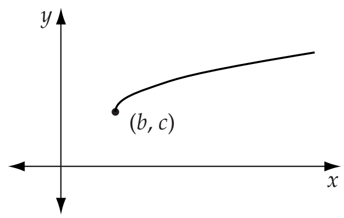
Truncus

- The standard equation of a truncus is
  $y = \dfrac{a}{(x-b)^2} + c,\ x \neq b$
  where: a is the dilation factor from the x-axis or parallel to the y-axis
  b is the horizontal translation
  c is the vertical translation
  when a is negative the standard graph has been reflected in the x-axis
- A truncus written in the general form has asymptotes of $x = b$ and $y = c$.



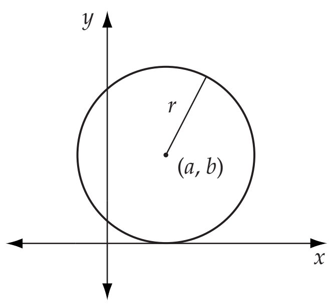
Square root function

- The general form of the square root function is $y = a\sqrt{x-b} + c$.
- The transformations undergone from $f(x) = \sqrt{x}$ are:
  a: dilation by a factor of a from the x-axis or parallel to the y-axis
  b: translation horizontally b units
  c: translation vertically c units.
- The domain of the equation is $x \geqslant b$ since the square root of a negative number cannot be found.
- The standard graph looks like:



Circles

- A circle is the set of points equidistant from a fixed point. It is not a function but a relation.
- The standard equation of a circle is $(x-a)^2 + (x-b)^2 = r^2$ where the centre of the circle is (a, b) and the radius is r.



- If the equation of a circle is not in this form it can be changed by completing the square in order to identify the centre and radius easily.
- When graphing circles on the CAS they have to be entered in two parts and the graph may not always be continuous.

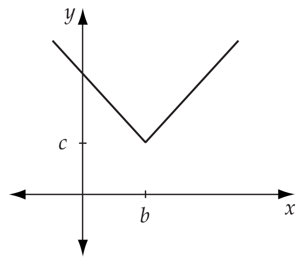
Inverse functions

- A function can only have an inverse function if it is a one-to-one function. Using the vertical line test and horizontal line test it is possible to determine if a function is one-to-one.
- If a function is not one-to-one then an inverse can be found but it is not an inverse function. The domain of the function can be limited so that an inverse function will exist.
- The inverse function is written as $f^{-1}(x)$.

- The inverse can be found by:
  – swapping the x and y and rearranging the equation, or
  – reflecting the graph of the function in the line $y = x$.
- CAS can be used to rearrange equations but care needs to be taken in interpreting the solution.
- Any points of intersection of the function and its inverse will lie on the line $y = x$ (i.e. they will have the same x and y coordinates).
- Domain of $f(x)$ is the range of $f^{-1}(x)$.
- Range of $f(x)$ is the domain of $f^{-1}(x)$.

Absolute value function

- The absolute value is the size of the variable regardless of its sign.
- The graph of an absolute value will have a sharp turning point if the graph of the original equation passed through the x-axis prior to taking the absolute value.
- To graph an absolute value function it is easiest to graph the part of the function inside the absolute value signs and then reflect any negative part in the x-axis. If the function includes other parts the appropriate transformations can then be made.
- The general equation for a linear absolute value function can be written: $y = a|x - b| + c$
- The graph of this function looks like:



Frequently asked questions

How do I know what domain to use when limiting a domain in order to find an inverse function?
Usually there are a number of options to use, so if a question doesn't specify which a number of answers will be correct, as long as the function for the domain used is one-to-one.

When I am graphing a function, how do I show dilation?
The easiest way to show that a function has been dilated is by specifying one or two points that are not intercepts. For example, if the original graph went through (1, 3) and it has been dilated by a factor of 2 from the x-axis, then the new graph will pass through (1, 6), so this point could be shown on the graph.

Study notes

- When determining the transformations that a function has undergone from its base function, always make sure that it is in standard form.
- If finding the inverse of a graph, reflect the graph in the line $y = x$, make the x-intercept of the function the y-intercept of the inverse, and vice versa.
- Use your CAS to assist with graphs but make sure that you know the basic shapes without it.

Cumulative Practice Examination 1

45 mins   Chapters 1–4   Total marks: 25

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

1 Find the equation of a line that passes through the point (-3, 2) and is perpendicular to the line $3x - 2y + 5 = 0$. (2 marks)

2 The turning point of a quadratic is (-2, 3) and the graph passes through the point (1, 5). Find the equation of the quadratic. (2 marks)

3 Given the function $y = -\frac{1}{3}(2x + 1)^3 + 5$:

(a) state the transformations that were performed on the function $y = x^3$ to achieve this function

(b) sketch the graph of y. (2 + 1 = 3 marks)

4 Find the equation of the graph for each of the following.

(a) a quartic that has a point of inflection at (4, 0), another x-intercept of (-2, 0) and a y-intercept at (0, 256)

(b) $f(x) = \sqrt{x}$ that has been dilated by a factor of 3 from the x-axis, reflected in the x-axis and translated 3 units right.

(c) a truncus with asymptotes at $x = 4$ and $y = 2$ that passes through (3, 4) (2 + 1 + 2 = 5 marks)

5 Solve each of the following.
(a) $(x-2)^2(x+3)(2x+1)=0$
(b) $(x^2-4)(x^2+7x+6)=0$
(c) $(x+1)(x^3-7x+6)=0$ (1 + 2 + 3 = 6 marks)

6 Draw the graph of each of the following.
(a) $(x-3)^2+(y+2)^2=4$
(b) $y=\dfrac{2}{x+3}-4$
(c) $y=|x-5|$ (1 + 2 + 1 = 4 marks)

7 (a) Rearrange $x^2-10x+y^2+6y+25=0$ into the standard form of an equation for a circle.
(b) Graph the circle. (2 + 1 = 3 marks)

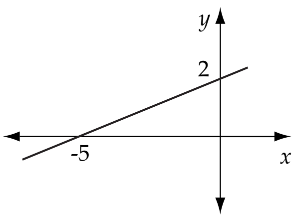
Cumulative Practice Examination 2

60 mins Chapters 1–4 Total marks: 37

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

Multiple choice

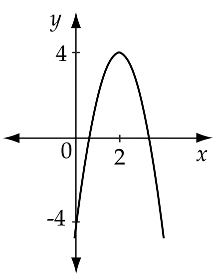
1 The equation of the line shown is:



A $y=-\frac{2}{5}x+2$ **B** $y=\frac{5}{2}x+2$ **C** $y=\frac{2}{5}x-5$
D $y=\frac{2}{5}x+2$ **E** $y=\frac{5}{2}x-5$

2 The value of k for which $x^2+kx-4=0$ has two distinct solutions is:
A $k=-16$ **B** $k=3$ **C** all values of k
D no value of k **E** $k>-4$

3 The equation of the graph shown would be:

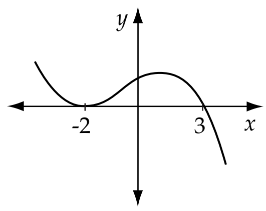


A $y=-2x^2-4x+2$ **B** $y=(x-2)^2+4$
C $y=-2x^2+8x-4$ **D** $y=-(x-2)^2+4$
E $y=-(x-2)^2-8$

4 The remainder when $2x^3+7x^2-5x+1$ is divided by $x-4$ is:
A 5 **B** 261 **C** -219 **D** 221 **E** 27

5 The range of $y=x^3+2x^2-11x+1$, $x\geqslant 3$, is:
A $y>13$ **B** R **C** $[13,\infty)$
D $[3,\infty)$ **E** $y>-7.74$

6 A possible equation for the following graph is:



A $y=(x+2)(x-3)$ **B** $y=(x+2)^2(3-x)$
C $y=(x+2)^2(x-3)$ **D** $y=(x+2)(3-x)^2$
E $y=-(x+2)^2(3-x)$

7 The function $f(x)$ has been transformed by being dilated by a factor of 2 from the x-axis and translated 3 units to the right. The transformed function would be:
A $-2f(x)+3$ **B** $f(2x+3)$ **C** $f(2x)+3$
D $2f(x+3)$ **E** $2f(x-3)$

8 The turning points of a quartic include one at (-2, 0) and another at (3, 0). The y-intercept is -36. Which of the following could be the equation?
A $y=-(x+2)^2(x-3)^2$ **B** $y=-36(x+2)^2(x-3)^2$
C $y=-(x-2)^2(x+3)^2$ **D** $y=-36(x-2)^2(x+3)^2$
E $y=-6(x+2)(x+3)^3$

9 The equation of a truncus with asymptotes of $x=1$ and $y=-2$ and a y-intercept at (0, -1) would be:
A $y=\dfrac{1}{(x-1)^2}-2$ **B** $y=\dfrac{2}{(x+1)^2}$
C $y=\dfrac{-1}{(x+2)^2}+1$ **D** $y=\dfrac{-2}{(x-1)^2}$
E $y=\dfrac{1}{(x+1)^2}+2$

10 For the function $f(x)=\sqrt{x+3}-2$, which of the following statements is incorrect?
A $f(x)$ doesn't have any asymptotes.
B The domain of the function is R.
C The inverse of the function $f(x)$ is $f^{-1}(x)=(x+2)^2-3$ with domain $x\geqslant -3$.
D The graph of $f(x)$ is the graph of $g(x)=\sqrt{x}$ translated 3 units to the left and 2 units down.
E $f(x)$ is not defined for $x=-5$

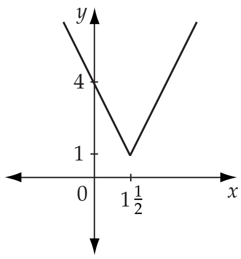
RETURN TO MAIN MENU

11 The equation $x^2 + y^2 = 4x + 10y + 16$ describes a circle with:

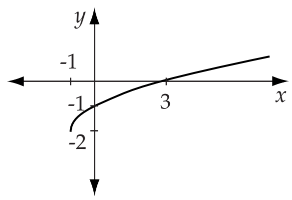
A centre (2, 5) and radius $3\sqrt{5}$
B centre (-2, -5) and radius $3\sqrt{5}$
C centre (2, 5) and radius 4
D centre (-2, -5) and radius 4
E centre (4, 10) and radius 4

12 For the graph shown, the best equation is:

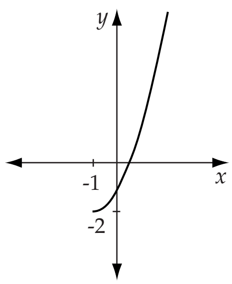
A $y = 2|x - 3|$
B $y = |2x - 3| + 1$
C $y = |x + 4|$
D $y = |-x + 4|$
E $y = |x + 3| + 1$



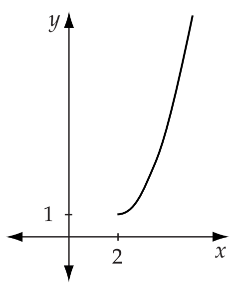
13 The graph of the inverse of this graph would be:



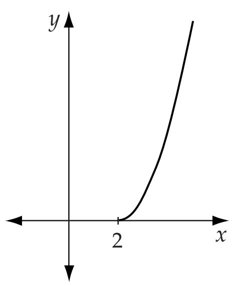
A



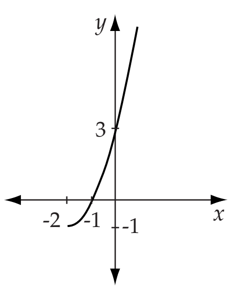
B



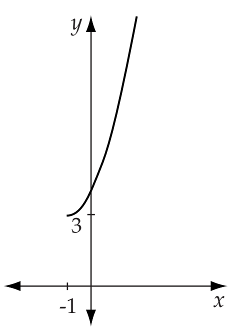
C



D



E



Extended answer

1 A quartic function, $f(x)$, has a turning point at (2, 0), x-intercepts of (3, 0) and (-2, 0), and passes through (4, 48).

(a) Find the equation of the function. (3 marks)

(b) Find the coordinates of the y-intercept. (1 mark)

(c) A straight line is drawn that passes through the y-intercept of $f(x)$ and has x-intercept at (2, 0). Find the equation of the line. (2 marks)

(d) Determine the coordinates of the points of intersection algebraically. (3 marks)

(e) Draw the graphs on the same set of axes, clearly showing the points of intersection. (3 marks)

2 (a) On the same set of axes sketch $f(x) = \dfrac{1}{x-3}$ and $h(x) = \dfrac{1}{(x-3)^2}$. (2 marks)

(b) State the value of the y-intercepts for $f(x)$ and $h(x)$. (1 mark)

(c) What is the exact value of the point of intersection of $f(x)$ and $h(x)$? (1 mark)

(d) Use algebra to find the exact value of the point of intersection for each of the following pairs of functions.

(i) $f(x) = \dfrac{1}{x-4}$ and $h(x) = \dfrac{1}{(x-4)^2}$

(ii) $f(x) = \dfrac{1}{x+2}$ and $h(x) = \dfrac{1}{(x+2)^2}$

(iii) $f(x) = \dfrac{1}{x+1}$ and $h(x) = \dfrac{1}{(x+1)^2}$

(2 + 2 + 2 = 6 marks)

(e) What general conclusion can you reach about the x value of the point of intersection for $f(x) = \dfrac{1}{x-a}$ and $h(x) = \dfrac{1}{(x-a)^2}$? (1 mark)

(f) Use your CAS to confirm this conclusion. (1 mark)

5. Probability and simulation

Summary

Probability

- Probability is the study of chance.
- Probability is the long-term relative frequency that an event will occur.
- The probability that a particular event will occur is determined by the rule probability of an event = $\dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes possible}}$
- Real-life events can be simulated by experiments. The results become more reliable as the number of trials increases.
- The study of probability is assisted by the use of Venn diagrams, set theory, tree diagrams and probability tables.
- The probability for any particular event, A, must lie in the interval $0 \leqslant \Pr(A) \leqslant 1$.
- The sum of probabilities for any particular situation must be equal to 1.

Notation

- Set notation is used extensively in probability. The main symbols, and their meanings, are:

ε	the universal set; the list of possibilities
A, B, C etc.	capital letters are used as set names
$n(A)$	the number of elements in the set A
$A \subset B$	A is a subset of B; every element of A is also an element of B
$A \subseteq B$	A is a subset of, or equal to, B; there might be no elements of B that are not also in A
$\in$	is an element of
$\cap$	the intersection between two sets; often indicated by use of the word 'and'
$\cup$	the union of two sets; often indicated by the use of the word 'or'
A'	the complement of the set A; all elements of ε not in A

Addition rule for probability

- The addition rule for probability holds that $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
- $\Pr(A') = 1 - \Pr(A)$

Conditional probability

- Conditional probability occurs when we can reduce the sample space due to some extra information. The rule for this is
$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)},\ \Pr(B) \neq 0$$
- This can be transposed to give the multiplication rule for probability:
$$\Pr(A \cap B) = \Pr(A \mid B) \times \Pr(B)$$
- If $\Pr(A \mid B) = \Pr(A)$, i.e. knowing something about event B has no effect on event A, we say that events A and B are independent. The formal definition of independence holds that $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$. This is a necessary and sufficient condition for independence.

Frequently asked questions

How many times do I need to run a simulation?

There is no hard and fast rule to determine the number of trials necessary for a simulation. However, the more times it is run, the more likely it is that the experimental results will match what theory would predict.

What is the difference between drawing cards, for instance, without replacement compared to drawing them with replacement?

The big difference is that when the cards are not replaced the probabilities change for each draw. When the cards are replaced, each draw is essentially a replica of every other draw and so has the same probabilities attached to it.

Study notes

- Conditional probability questions need careful analysis. The important thing to remember is that the condition that is imposed effectively reduces the size of the universal set. Just find the reduced universal set and then things should proceed quite smoothly.
- Venn diagrams and tree diagrams are very useful tools to use in this area. In many questions such a diagram should be drawn even if the question does not specifically ask for it. The picture makes the identification of sets and outcomes much clearer.
- The rule $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$, which applies to independent events, is one of the most frequently used rules in this area of mathematics. Frequently there is no formal mention that the events are independent. In many cases you need to rely on your common sense to determine whether or not the events are independent.

RETURN TO MAIN MENU

Cumulative Practice Examination 1

45 mins | Chapters 1–5 | Total marks: 25

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

1 Given $y = -2x^2 - 12x - 13$:

(a) express y in turning point form

(b) state the maximum value of y.

(2 + 1 = 3 marks)

2 Expand the following.

(a) $(3x - 5)^3$ **(b)** $\left(\dfrac{2x}{3} + \dfrac{6}{x^2}\right)^3$

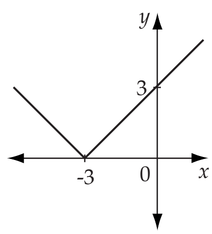
(2 + 2 = 4 marks)

3 Find the inverse for the following function.

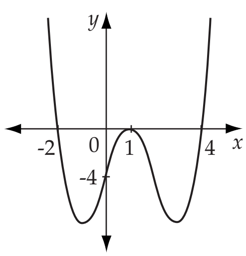
$y = \dfrac{3}{x+2} - 4$ (2 marks)

4 Find the equation for each of the following graphs.

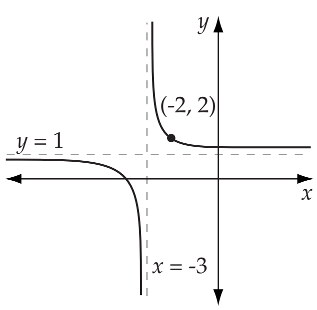
(a)



(b)



(c)



(1 + 1 + 1 = 3 marks)

5 A twenty-sided die marked 1–20 is rolled. Find the probability that the number showing uppermost is:

(a) a multiple of five

(b) a factor of 20

(c) a multiple of five or a factor of 20

(d) a multiple of five but not a factor of 20

(1 + 1 + 1 + 1 = 4 marks)

6 Joan, Joyce and Julie love playing darts. The probabilities that each will score a triple twenty with any particular shot are $\frac{2}{3}$, $\frac{3}{5}$ and $\frac{3}{4}$ respectively. Find the probability that:

(a) all three score a triple twenty

(b) at least one scores a triple twenty

(c) exactly two of them score a triple twenty

(d) only Joan scores a triple twenty

(1 + 1 + 2 + 1 = 5 marks)

7 There are two packs of 52 playing cards, Pack A and Pack B. A card is drawn at random from Pack A and if it is a heart another card is drawn from Pack A; otherwise the second card is drawn from Pack B.

(a) Find the probability that both cards are hearts.

(b) Find the probability that both cards are clubs.

(c) Find the probability that only one of the cards is a heart.

(d) Find the probability that the second card is a club if the first was a heart.

(1 + 1 + 1 + 1 = 4 marks)

Cumulative Practice Examination 2

60 mins | Chapters 1–5 | Total marks: 37

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

Multiple choice

1 The graph of $y = x^2 + 6x + m$ will have a minimum turning point at (-3, 0) if:

A $m = 3$ **B** $m = 6$ **C** $m = 9$

D $m = -3$ **E** $m = -9$

2 The expansion of $(6 - 5x)(5x + 6)(3x - 2)$ is:

A $75x^3 + 50x^2 - 108x + 72$

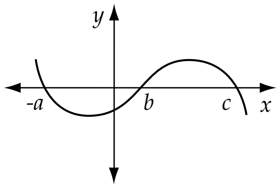
B $-75x^3 + 50x^2 + 108x - 72$

C $75x^3 - 130x^2 + 228x - 72$

D $-75x^3 + 130x^2 + 228x - 72$

E $-75x^3 + 50x^2 + 12x + 72$

3 If a, b and c are positive numbers, then a possible equation for the graph shown is:



A $y = (x - a)(x - b)(x - c)$
B $y = (x + a)(x - b)(x - c)$
C $y = (x - a)(x + b)(x + c)$
D $y = (a - x)(x - b)(x - c)$
E $y = (x + a)(b - x)(x - c)$

4 If all the solutions of an equation are $x = 2$, -2 and 3, the equation could be:
A $x^4 - 2x^3 - 7x^2 - 8x + 12 = 0$
B $x^4 + 2x^3 - 2x^2 + 3x = 0$
C $x^4 - 5x^3 + 2x^2 + 20x - 24 = 0$
D $(x - 2)(x + 2)^3(x - 3) = 2$
E $(x + 2)(x - 3)(x + 1)(x - 2) = 0$

5 The graph of $f(x) = \dfrac{-2}{x+3} + 1$ can be found by performing which of the following transformations to $g(x) = \dfrac{1}{x}$?
A reflection in the y-axis, dilation by a factor of $\frac{1}{2}$ from the x-axis, translation of 3 units to the left and 1 unit up.
B reflection in the y-axis, dilation by a factor of 2 from the x-axis, translation of 3 units to the right and 1 unit up.
C reflection in the x-axis, dilation by a factor of $\frac{1}{2}$ from the x-axis, translation of 1 unit to the right and 3 units down.
D reflection in the y-axis, dilation by a factor of 2 from the x-axis, translation of 1 unit to the right and 3 units down.
E reflection in the x-axis, dilation by a factor of 2 from the x-axis, translation of 3 units to the left and 1 unit up.

6 The graph of the equation $x^2 + 4x + y^2 - 6y = -4$ is:
A a circle with radius 2 units and centre (4, -6)
B a circle with radius 2 units and centre (2, 3)
C a circle with radius 3 units and centre (4, -6)
D a circle with radius 3 units and centre (-2, 3)
E a circle with radius 4 units and centre (1, -1)

7 A spinner is divided into five unequal coloured sections: red, blue, green, orange and pink. It was spun fifty times and the results obtained were:

Colour	red	blue	green	orange	pink
Number recorded	7	15	12	9	7

Based on these results, the probability that the next spin will not result in green is:
A $\frac{4}{5}$ **B** $\frac{19}{25}$ **C** $\frac{1}{5}$ **D** $\frac{13}{19}$ **E** $\frac{6}{25}$

8 An egg packing company knows from past experience that one-sixth of all eggs pass through the inspection point with minor flaws. A simulation was run where H1 and T1 represented flawed eggs. Sixty trials were conducted with the following results.

H4	H1	T1	H4	H6	H3	T2	T1	H1	T3	T2	H4
T3	T4	H4	T1	H5	T2	T6	H2	T2	H3	H6	T3
T1	H1	H3	T2	H3	T1	H5	T4	T6	H1	H1	H6
T4	T4	H6	T5	H6	T4	T4	H1	H6	T2	T4	H3
T2	H6	T3	T5	H6	T2	H3	T4	T6	H6	T1	H5

Each row represents a dozen eggs.
Based on these results, the probability that a dozen eggs contains no more than one egg with a minor flaw is:
A $\frac{2}{5}$ **B** $\frac{1}{5}$ **C** $\frac{4}{5}$ **D** $\frac{1}{6}$ **E** $\frac{3}{5}$

9 The twelve letters in the word HIPPOPOTAMUS are placed in a box. The probability that the first letter drawn out of the box is a vowel is:
A $\frac{1}{2}$ **B** $\frac{4}{9}$ **C** $\frac{1}{3}$ **D** $\frac{1}{12}$ **E** $\frac{5}{12}$

The information below applies to Questions **10** and **11**.
$\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$,
$A = \{1, 4, 9, 16\}$, $B = \{1, 2, 4, 8, 16\}$

10 $n(A' \cap B)$ is equal to:
A 2 **B** 3 **C** 5 **D** 10 **E** 13

11 $(A \cap B)'$ is equal to:
A {1, 4, 16} **B** {1, 2, 4, 8, 9, 16}
C {2, 8, 9}
D {2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
E {3, 5, 6, 7, 10, 11, 12, 13, 14, 15}

12 If $\Pr(A \cap B) = 0.3$, $\Pr(B') = 0.25$ and $\Pr(A) = 0.45$, then $\Pr(A' \cap B')$ is equal to:
A 0.55 **B** 0.1 **C** 0.45 **D** 0.15 **E** 0.75

13 A spinner is equally divided into five pieces, two of which are green, one blue, one red and one yellow. If green comes up on the spinner then a fair coin is tossed. The probability of the coin being tossed and resulting in a head is:
A $\frac{2}{5}$ **B** $\frac{9}{10}$ **C** $\frac{1}{10}$ **D** $\frac{1}{2}$ **E** $\frac{1}{5}$

Extended answer

1 **(a)** Find the equation of the quadratic function $f(x)$ that passes through (0, -3), (1, 5) and (2, 15). (2 marks)
(b) Find the x-intercepts exactly. (2 marks)
(c) Draw the graph, clearly identifying all significant points. (2 marks)
(d) Find the equation of the inverse relation of $f(x)$. (2 marks)
(e) By limiting the domain of $f(x)$, sketch a graph of the inverse function of $f(x)$ and state its domain and range. (2 marks)

(f) Find the point(s) of intersection of the function and its inverse (giving your answer correct to two decimal places). (1 mark)

2 Class A consists of 12 boys and 10 girls and Class B consists of 15 boys and 9 girls.

(a) What is the probability of a student randomly selected from Class A being a girl? (1 mark)

(b) What is the probability of a student randomly selected from Class B being a girl? (1 mark)

Assume we first pick one of the classes at random and then choose the student.

(c) What is the probability that this student is a girl? (2 marks)

(d) Is this answer the same as would be obtained if we treated the two classes as one big class? (1 mark)

(e) Explain why this is the case. (2 marks)

Now consider Class C that has 12 boys and 12 girls and Class D that has 11 boys and 11 girls.

(f) What is the probability of a student randomly selected from Class C being a girl? (1 mark)

(g) What is the probability of a student randomly selected from Class D being a girl? (1 mark)

Assume we randomly pick the class before we select the student.

(h) What is the probability that this student is a girl? (2 marks)

(i) Is this answer the same as would be obtained if we treated the two classes as one big class? (1 mark)

(j) Explain why this is the case. (1 mark)

RETURN TO MAIN MENU

6. Calculus

Summary

Rates of change

- Rates are used to describe quantities that are changing.
- The rate of change of y with respect to x is written as:

$$\text{rate} = \text{gradient} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

- Rates of change of linear functions are constant.
- If a quantity does not change in value then its rate of change is equal to zero.
- The gradient or rate of change of a curve (other than a straight line) varies and is dependent upon the value of x at which the rate is being measured.

Average rate of change

- The average rate of change of a function between two points is equal to the gradient of the chord connecting the two points that lie on the curve.

Instantaneous rate of change

- The instantaneous rate of change equals the gradient of the tangent to the curve at the point at which the rate is being measured.
- To sketch the graph of the gradient function given the graph or equation describing y:

Graph of y	*Graph of* $\frac{\Delta y}{\Delta x}$
Positive gradient	Sketch above the x-axis
Negative gradient	Sketch below the x-axis
Turning point (gradient = 0)	x-intercepts

Displacement and velocity

- Distance (d) describes the total distance travelled by a body, without consideration of the direction of motion.
- Displacement (s) describes how far the body has travelled from a fixed point called the origin.
- Speed describes the rate of change of the distance travelled with respect to time.
- Velocity (v) describes the rate of change of the position of a body with respect to time.
- The gradient of a displacement–time graph represents the velocity of the particle.

Continuity and hybrid functions

- A function is said to be continuous if its graph can be drawn without lifting your pen off the paper across the specified domain.
- A function is discontinuous in a region of a graph that has holes, breaks or jumps.
- Open circles (○) indicate that the function is undefined or discontinuous at that point.
- Closed circles (●) indicate that the function is defined or continuous at that point.
- When sketching graphs of functions:
  - If the given inequality is $\leqslant$ or $\geqslant$, use a closed circle (●).
  - If the given inequality is $<$ or $>$, use an open circle (○).
- A hybrid function is a function for which different rules or equations apply to the different parts of its domain.

Limits

- The limit describes the behaviour of y as the curve gets closer to a particular value of x.
- A limit will exist at $x = a$ if the left-hand limit is equal to the right-hand limit.
- Limits may be evaluated algebraically, graphically or by numerical substitution.
- If the points $A\,(x, f(x))$ and $B\,((x + h), f(x + h))$ lie on the curve $f(x)$, then:
  - the gradient of the chord AB or the average rate of change is given by $\dfrac{f(x+h) - f(x)}{h}$
  - the gradient of the tangent or the instantaneous rate of change at A is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Differentiation

- If y is a function of x then the derivative is denoted as $\dfrac{dy}{dx}$.
- If $f(x)$ is a function of x then the derivative is denoted as $f'(x)$.
- To find a derivative, multiply each term by the power of x and then lower the power by one.
  If $f(x) = ax^n$ then $f'(x) = anx^{n-1}$
- The derivative of a constant is equal to 0.
  If $f(x) = k$ then $f'(x) = 0$
- The derivative of a sum/difference of terms is equal to the sum/difference of the derivatives of each term.
  If $f(x) = g(x) \pm h(x)$ then $f'(x) = g'(x) \pm h'(x)$

Antidifferentiation

- Antidifferentiation is the opposite process to differentiation:

$$\begin{tikzcd} f(x) \arrow[r, "\text{differentiation}"] & f'(x) \\ y & \frac{dy}{dx} \arrow[l, "\text{antidifferentiation}"'] \end{tikzcd}$$

- To find an antiderivative, raise the power of x by one and then divide the term by the new power.
  $\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + c,\ n \neq -1$
- The antiderivative of a sum/difference is equal to the sum/difference of the antiderivatives of each term.
  $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Techniques to simplify equations before differentiating or antidifferentiating

- Expand brackets.
- Apply index laws.
- Factorise and eliminate terms by cancellation.
- If there is only one term in the denominator, rewrite each term in the numerator over the denominator and simplify.
- Always bring terms involving x in the denominator to the numerator by changing the sign on the power.
- Express answers with positive powers.
- If terms involving x are present in the denominator, remember to state restrictions on the values of x.

Tangents and normals

- If P is a point on a curve $y = f(x)$ with coordinates (x_1, y_1) then the equation of the tangent at (x_1, y_1) is found by $y - y_1 = f'(x_1)(x - x_1)$.
- The equation of the normal at this point is found by
  $y - y_1 = \frac{-1}{m}(x - x_1)$ where $m = f'(x_1)$.

Stationary points

- A stationary point on a graph will occur when $f'(x) = 0$.
- The nature of a stationary point can be determined by the sign of the gradient immediately on either side of the point.

$x < a$	$x = a$	$x > a$	*Nature of stationary point*
+	0	−	local maximum
−	0	+	local minimum
+	0	+	stationary point of inflection
−	0	−	stationary point of inflection

- To solve maximum/minimum problems:
  1. Draw a diagram or establish a connection using as few variables as possible.
  2. Express the quantity that needs to be maximised/minimised in terms of one variable.
  3. Differentiate, let the derivative equal zero, and solve.
  4. Check the nature of the point.
  5. Answer the actual question, i.e. dimension, area, volume etc. (Remember to consider the reality of the situation—you can't have negative time, volume etc.)

Rates of change problems

- For rate of change problems there are two main situations:
  – A quantity is given and there are questions about the rate. Differentiate and substitute the value into the rate equation.
  – A rate is given and there are questions about the actual quantity. Antidifferentiate, find the constant of antidifferentiation, and then substitute the values into the equation describing the quantity.

Frequently asked questions

When do I use $\frac{rise}{run}$ and when do I use the gradient of the tangent?

Use $\frac{\text{rise}}{\text{run}}$ or the gradient of a chord for the average rate of change between two points; use gradient of the tangent to find instantaneous rate of change at a point.

If the graph shows a flat line, what is the rate of change?

A horizontal flat line means zero rate of change, and a vertical line means the rate of change is undefined.

What is the purpose of differentiation by first principles, and do I have to know how to do it?

Differentiation from first principles shows the basis of finding the gradient of a tangent to a curve at a point. It is important to know how to do it, but unless told to use it, differentiation by rule is always easier.

How do I know when to integrate and when to differentiate?

When integrating you are given the gradient function and asked to find the function. When differentiating you are finding the gradient or the rate of change of a function. It is always important to look at the question and the symbols carefully.

Study notes

- When looking at a graph always check the labels on the axes.
- The word average implies $\frac{\text{change in } y \text{ values}}{\text{change in } x \text{ values}}$.
- Always remember that a function is not differentiable at an endpoint even if that endpoint is included in the function.
- Look at the gradient either side of a stationary point to determine what sort of stationary point it is.
- Not all points of inflection are stationary points.
- Don't forget the constant of integration when antidifferentiating.

RETURN TO MAIN MENU

Cumulative Practice Examination 1

45 mins | Chapters 1–6 | Total marks: 25

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

1 Solve the following sets of simultaneous equations.

(a) $3x - 5y = 21$
$y = x - 5$

(b) $2y + 3x = 7$
$5y - 3x = -14$

(3 + 2 = 5 marks)

2 The 855 students at Top Results College were surveyed regarding their opinion about wearing school uniform. The results are displayed in the following table.

	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	
In favour	80	85	60	50	45	35	355
Against	70	75	80	95	80	65	465
Not sure	10	5	10	5	5	0	35
	160	165	150	150	150	130	855

Give your answers to these questions as simplified fractions.

(a) What is the probability that a student selected at random from the school population will be in favour of wearing school uniform? (1 mark)

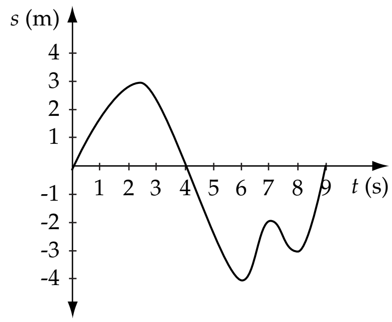
(b) What is the probability that a randomly selected Year 8 student is against wearing school uniform? (1 mark)

(c) What is the probability that a randomly selected Year 11 student is not sure on this matter? (1 mark)

(d) A student is selected at random from the school population. Given that the student is from Year 7, what is the probability that this student is in favour of wearing school uniform? (2 marks)

(1 + 1 + 1 + 2 = 5 marks)

3 Use the displacement–time graph shown to determine the total distance travelled.



(2 marks)

4 Find the derivative of $f(x) = x^2 + 3$ using first principles. (2 marks)

5 Find the equation of the tangent to the curve $y = 4x^2 + 5x + 3$ when $x = 2$. (3 marks)

6 **(a)** Find the coordinates of the turning points of the curve $y = x^3 + 2x^2 - 4x + 1$.

(b) Hence sketch the curve. (Do not calculate x intercepts.) (3 + 2 = 5 marks)

7 If $f'(x) = 3x^2 + 4x + 2$, find the equation of the function $f(x)$ that passes through the point (1, 2). (3 marks)

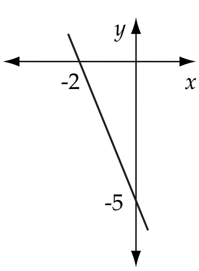
Cumulative Practice Examination 2

90 mins | Chapters 1–6 | Total marks: 52

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

Multiple choice

1 The equation of the line shown is:



A $y = -2x - 5$
B $y = \frac{5}{2}x - 5$
C $y = -\frac{5}{2}x - 5$
D $y = -\frac{2}{5}x - 5$
E $y = \frac{5}{2}x - 2$

2 A factor of $2z^3 + 15z^2 + 22z - 15$ is:

A $z + 1$ **B** $z + 3$ **C** $z - 2$ **D** $z - 4$ **E** $z - 5$

3 The relationship between x and y is best modelled by:

x	2	5	8	11
y	-9	42	327	1008

A a linear model
B a cubic model
C a quadratic model
D a hyperbolic model
E none of these

4 The highest common factor of the expression $16x^4 + 8x^2 + 32xy$ is:

A $16x^2$ **B** $8x$ **C** 2 **D** $4x^2$ **E** x

5 If we have two events P and R such that $\Pr(P) = \frac{2}{3}$, $\Pr(R) = \frac{1}{5}$ and $\Pr(P \cup R) = \frac{3}{4}$, then $\Pr(P \cap R)$ is equal to:

A $\frac{13}{15}$ **B** $\frac{11}{20}$ **C** $\frac{1}{12}$ **D** $\frac{2}{15}$ **E** $\frac{7}{60}$

6 A fair coin is tossed and then a normal die is rolled. The probability of getting a tail followed by a multiple of 3 is:

A $\frac{1}{6}$ **B** $\frac{5}{6}$ **C** $\frac{2}{3}$ **D** $\frac{2}{5}$ **E** $\frac{1}{5}$

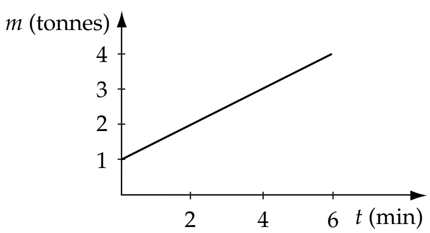
7 The average rate of flow in L/min of a tap that fills a 40 litre drum in 8 minutes is:

A 8 L/min **B** 5 L/min
C 10 L/min **D** 13 L/min
E 0.2 L/min

8 A car travels at a constant speed between points A and B. At point A the time is 2 hours since starting, and the car is 130 km from the starting point. It arrives at point B half an hour later, and is 160 km from the starting point. What is the average speed travelled between A and B?

A 60 km/h **B** 65 km/h **C** 64 km/h
D 116 km/h **E** 80 km/h

9 The graph shows the change in mass of a truck, m tonnes, as it is filled with sand.

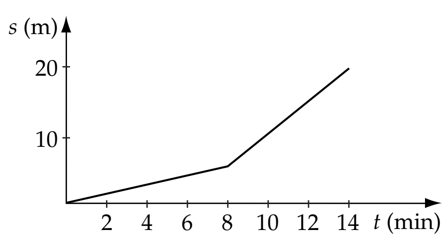


The rate of change in m in tonnes/min is:

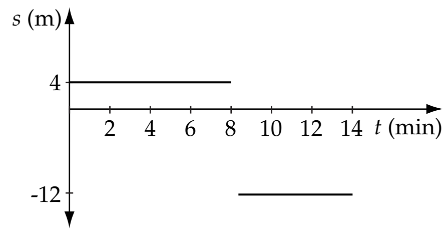
A 2 tonnes/min **B** 4 tonnes/min
C 0.5 tonnes/min **D** 1 tonne/min
E 1.5 tonnes/min

10 A particle moves north for 8 minutes at a constant speed for 4 metres, and travels a further 12 metres in a southerly direction for 6 minutes. Assuming north to be positive, the displacement–time graph would look like:

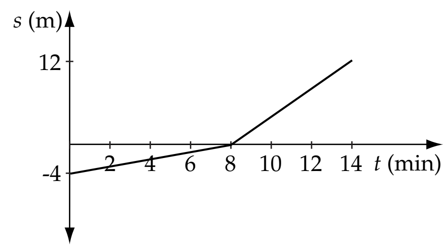
A



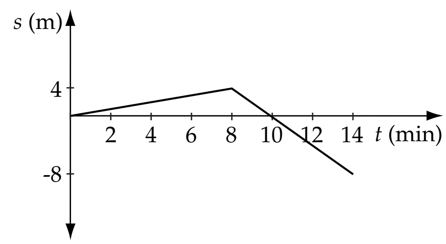
B



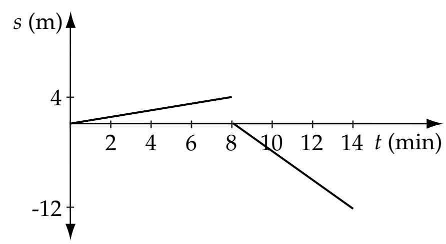
C



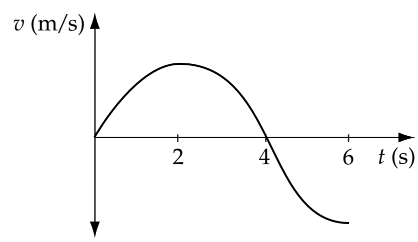
D



E



11 From the graph of the journey of a particle, at what times is the velocity decreasing?



A $2 < t < 6$ **B** $t > 4$ **C** $2 < t < 4$
D $t < 4$ **E** $t < 2$

12 The equation of the normal to the curve $y = x^3 + \sqrt{x}$ at $x = 1$ is:

A $2y - 7x + 3 = 0$ **B** $y = \dfrac{7x}{2} - \dfrac{11}{2}$
C $2x - 7y + 12 = 0$ **D** $y = 2x - 5$
E $2x + 7y - 16 = 0$

13 The gradient of the tangent of $f(x) = 3x^4 - \dfrac{2}{x}$ at $x = 3$ is:

A $324\frac{2}{9}$ **B** $323\frac{7}{9}$ **C** $2916\frac{2}{9}$
D $2915\frac{2}{9}$ **E** $-\frac{9}{2918}$

RETURN TO MAIN MENU

14 $\int \sqrt{x} + x^2 \, dx$ is equal to:

A $\dfrac{1}{2\sqrt{x}} + 2x + c$ **B** $x^3 + 2x + c$

C $\dfrac{2(\sqrt{x})^3 + x^3}{3} + c$ **D** $\dfrac{3x^{\frac{3}{2}}}{2} + x^3 + c$

E $x^{\frac{3}{2}} + x^3 + c$

15 Evaluating $\lim\limits_{x \to 4} \dfrac{3x^2 - 11x - 4}{x - 4}$ gives:

A 0 **B** 1 **C** 4
D 13 **E** limit does not exist

16 The gradient of the line joining the points $A(5, f(5))$ and $B(5 + h, f(5 + h))$ is:

A $\lim\limits_{h \to 0} \dfrac{f(5+h) - f(5)}{h}$ **B** $\dfrac{f(5+h) - f(5)}{5}$

C $\dfrac{f(5) - f(5+h)}{h}$ **D** $\dfrac{f(5) + f(5+h)}{h}$

E $\dfrac{f(5) - f(5+h)}{-h}$

17 The value of x at which the gradient of the tangent to the curve $f(x) = x^3 - 3x + 4$ is 6 is:

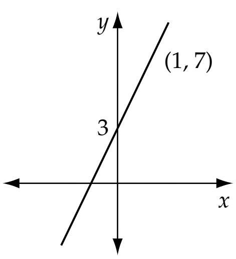
A $x = \pm\sqrt{3}$ **B** $x = \pm 3$ **C** $x = 0$
D $x = 2$ **E** $x = \pm 2$

18 The curve with equation $y = ax^2 + bx + c$ has a y-intercept of 22 and a turning point at (3, 4). The values of a, b and c respectively are:

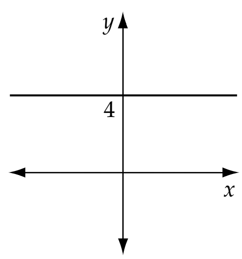
A 3, 4, 22 **B** 2, -12, 22 **C** 22, 3, 4
D 9, 16, 22 **E** 6, -18, 22

19 If $f'(x) = 4x + 3$ then the graph of $f(x)$ could be:

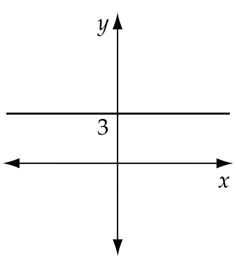
A



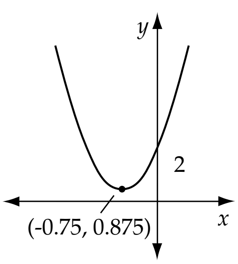
B



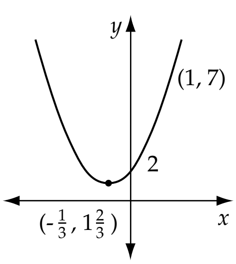
C



D



E



20 If the rate of change in volume in a container is given by $\dfrac{dV}{dt} = 3t^2 + 4t + 7$ and there is 15 cm^3 in the container when $t = 1$, then:

A $V = t^3 + 2t^2 + 7t + 5$ **B** $V = t^3 + 2t^2 + 7t + 5$
C $V = 6t + 4$ **D** $V = 3t^3 + 4t^2 + 7t + 1$
E $V = t^3 + 4t^2 + 7t + 3$

21 The graph of $f(x) = \sqrt{x}$ is translated 3 units to the left and 4 units down. The equation of the transformed graph is:

A $f(x) = \sqrt{x+3} - 4$ **B** $f(x) = \sqrt{x-4} - 3$
C $f(x) = \sqrt{x+4} - 3$ **D** $f(x) = \sqrt{x+3} + 4$
E $f(x) = \sqrt{x-3} - 4$

22 The inverse function for $f(x) = \dfrac{2}{x+3} - 1$ is:

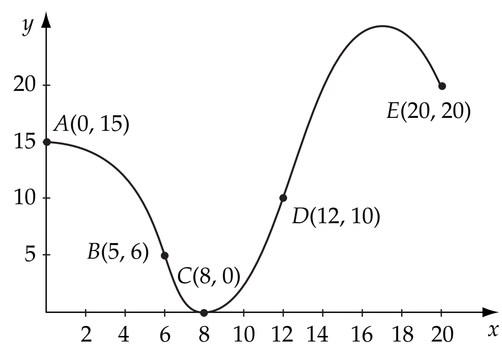
A $f^{-1}(x) = \dfrac{2}{x+1} - 3$ **B** $f^{-1}(x) = \dfrac{1}{x-2} - 3$

C $f^{-1}(x) = \dfrac{-3}{x+1} + 2$ **D** $f^{-1}(x) = \dfrac{2}{x-1} + 3$

E $f^{-1}(x) = \dfrac{-3}{x+2} + 1$

Extended answer

1 A section of the rollercoaster ride at Finnigan's Fun World need replacing. The track replacement cost is \$38 per metre of track and Finnigan wants to determine the approximate cost of the repairs. The section of track to be replaced is as follows:



Finnigan decides that the track can be modelled by three smoothly joined parabolas. For the curves to be smoothly joined they must have the same gradient at the point of contact.

(a) The general form of a parabola is $y = ax^2 + bx + c$. Find $\dfrac{dy}{dx}$, the gradient of a parabola at any point. (1 mark)

(b) Find the equation of the parabola passing through the points $B(5, 6)$, $C(8, 0)$ and $D(12, 10)$, using an algebraic approach. Use exact values to express your answer. Check your result using QuadReg on your CAS. (3 marks)

(c) **(i)** Find the gradient of the curve found in part **(b)** at the point $B(5, 6)$. (1 mark)

(ii) Hence find the equation of the parabola that passes through the points $A(0, 15)$ and $B(5, 6)$. (2 marks)

(d) Find the equation of the parabola that passes through the points $D(12, 10)$ and $E(20, 20)$. (3 marks)

(e) Verify the accuracy of your calculations by generating the section of track to be replaced using your CAS. Describe how you checked the accuracy. (1 mark)

The distance, d, between two points (x_1, y_1) and (x_2, y_2) is given by: $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

(f) Using 1 metre intervals between points, estimate the length of the rollercoaster track to be replaced and hence find an estimate of the replacement cost. (4 marks)

2 One particle follows the path given by $s(t) = t^2 + 4t - 5$ and another particle follows the path given by $m(t) = -t^2 + 3t + 4$; for both particles $0 \leqslant t \leqslant 6$.

(a) Find the turning point of each of the two paths. (3 marks)

(b) Hence find the maximum displacement reached by each of the particles. (2 marks)

(c) If the paths of the two particles intersect, find the time(s) that this occurs. (2 marks)

(d) Which particle travels the greatest distance? (1 mark)

3 Boxes of a certain breakfast cereal contain a plastic model of one of five different action heroes: ActionMan, TuffGirl, Muscles, Gonzo, and Predator. The same number of each figure were manufactured and placed in the cereal boxes. Tyler has collected four of the figures but is missing Muscles. Find the probability that:

(a) the next packet contains Muscles (1 mark)

(b) Muscles has still not appeared after three more packets (1 mark)

(c) it takes exactly four more packets to get Muscles (1 mark)

(d) it takes no more than four more packets to get Muscles (2 marks)

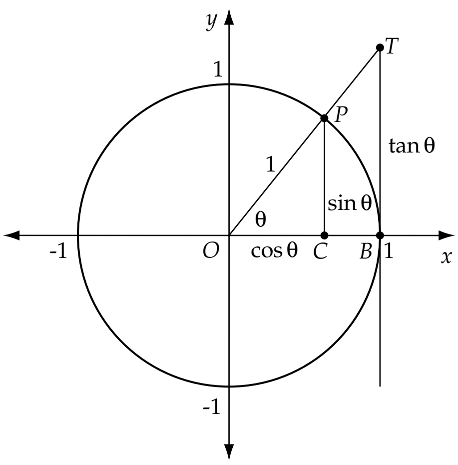
(e) Tyler gets three more Predators and then gets Muscles. (2 marks)

7. Circular functions

Summary

The unit circle

- The circular functions $\sin\theta$, $\cos\theta$ and $\tan\theta$ are defined in the unit circle as follows:



Trigonometric identities

- The relationships between the functions include the identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

- In a right-angled triangle, we can use the relationships:

$$\sin\theta = \frac{\text{O}}{\text{H}} \quad \cos\theta = \frac{\text{A}}{\text{H}} \quad \tan\theta = \frac{\text{O}}{\text{A}}$$

Radians

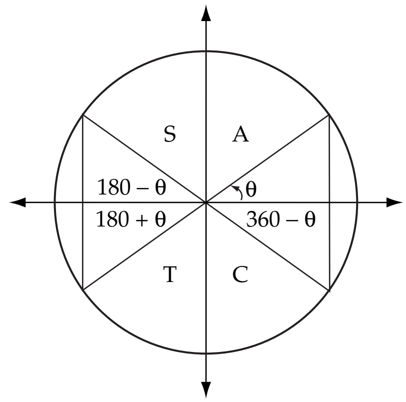
- Angles are preferably measured in radians. Since $\pi^c = 180°$, then

$$1^c = \frac{180°}{\pi} \text{ and } 1° = \frac{\pi^c}{180}$$

-

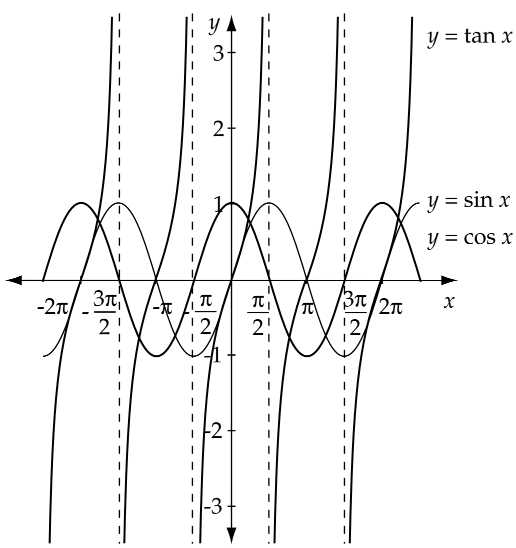
θ	0°	30°	45°	60°	90°
θ	0^c	$\frac{\pi^c}{6}$	$\frac{\pi^c}{4}$	$\frac{\pi^c}{3}$	$\frac{\pi^c}{2}$
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

- The second quadrant equivalent angle is $\pi - \theta$
- The third quadrant equivalent angle is $\pi + \theta$
- The fourth quadrant equivalent angle is $2\pi - \theta$



Trigonometric graphs

- The graph of $y = a \sin nx + k$ has an amplitude of $|a|$. If a is negative, the graph is reflected in the x-axis. Its median value is k, and its period is $\frac{2\pi}{n}$.
- The graph of $y = a \cos nx + k$ has an amplitude of $|a|$. If a is negative, the graph is reflected in the x-axis. Its median value is k, and its period is $\frac{2\pi}{n}$.
- The graph of $y = a \tan x$ has asymptotes wherever $\tan x$ is undefined. The first two positive asymptotes are the lines $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$. Its period is $\frac{\pi}{n}$.



RETURN TO MAIN MENU

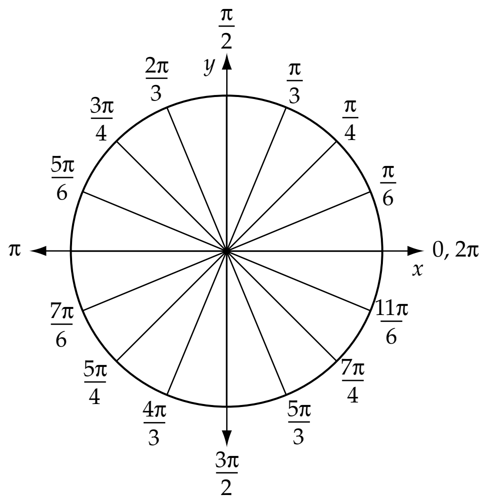
Frequently asked questions

Does it matter which I use: radians or degrees?
Yes, it is very important to check the mode on your CAS and always to take care with whether degrees or radians are being used, as the answers will be very different. If there is no degree sign then radians can be assumed.

What is the easiest way to draw a sine or cosine graph?
When you have determined the period, then use quarters of this to put points at the maximum, minimum and the points that lie on the median. Joining these with a smooth curve is quite easy.

Study notes

- When drawing a graph, always use a pencil and take care to make the curve smooth without any extra little sketch lines.
- Always leave your CAS in radian mode so that you know that if a question is in degrees you need to change it, and then remember to change it back.
- Sketch graphs, even when not required, can be helpful in understanding what a question is asking for.
- A useful diagram is:



Cumulative Practice Examination 1

60 mins Chapters 1–7 Total marks: 34

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

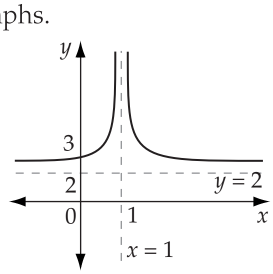
1 Given $\frac{ax + by}{c} = x - b$:

(a) make x the subject

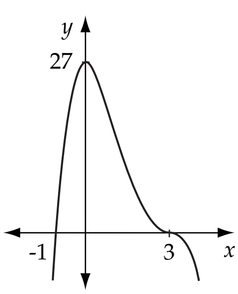
(b) find the value of y given $x = 4$, $b = 2$, $c = -3$ and $a = 1$. (3 + 2 = 5 marks)

2 Determine the equation for each of the following graphs.

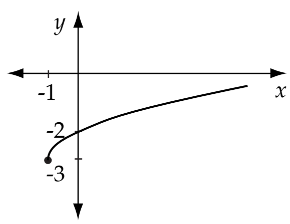
(a)



(b)



(c)



(1 + 2 + 1 = 4 marks)

3 A bag has 20 marbles numbered 1, 2, 3…20. One marble is drawn at random. Find the probability that the number on the marble is:

(a) odd

(b) greater than 8

(c) even given that it is greater than 7

(1 + 1 + 2 = 4 marks)

4 A tank starts with 45 litres of petrol. The petrol is being used at a constant rate of 9 litres for every 72 km.

(a) Write an equation to represent this situation.

(b) How many litres would be used to travel 216 km?

(c) After how many kilometres will the 45 litres of petrol be used up? (2 + 1 + 1 = 4 marks)

5 Differentiate the following by rule.

(a) $y = 4x^3 + 2x^2 + 5x + 6$

(b) $f(x) = \dfrac{3}{x^2}$

(c) $f(x) = (x + 1)(x^2 - 3x + 4)$

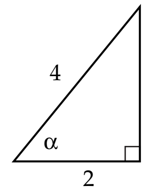
(d) $f(x) = \dfrac{x^3 + 4x}{x}$ (1 + 1 + 2 + 2 = 6 marks)

6 Sketch the graph of $y = -\cos 3x + 2$ over the domain $[0, \pi]$. (2 marks)

7 Find all the solutions for $\sin 2x = \dfrac{\sqrt{3}}{2}$ over the domain $[0, 2\pi]$. (3 marks)

8 A sine graph has a maximum value of 6 and a minimum value of -1. Its period is 12. Write down its possible equation. (2 marks)

9 For the following triangle:



(a) write down the exact value of sin α and cos α

(b) hence show that $\sin^2 \alpha + \cos^2 \alpha = 1$

(c) find the exact value of α. (2 + 1 + 1 = 4 marks)

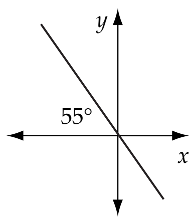
Cumulative Practice Examination 2

90 mins Chapters 1–7 Total marks: 52

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

Multiple choice

1 The gradient of the given line is:



A 55°
B tan 55°
C 125°
D -tan 125°
E -tan 55°

2 Which of the following transformations of $y = x^2$ would result in the graph of $y = 2x^2 + 4x$?

A dilation by 2, translation down 2
B dilation by 4 and translation left 2
C dilation by 3, translation left 1 and down 2
D dilation by 2, translation right 1 and down 2
E translation left 1 and down 1

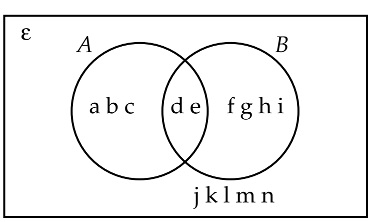
3 One of the factors of $x^3 - 10x^2 + 13x + 24$ is:

A $x - 1$ **B** $x + 8$ **C** $x - 8$
D $x + 3$ **E** $x - 2$

4 The range of the function $y = -\frac{1}{2}(5 - x)^3 + 3, x < 0$ is:

A $(-\infty, 0)$ **B** $(-\infty, -\frac{119}{2}]$ **C** $y < \frac{1}{2}$
D $y > -\frac{119}{2}$ **E** $(-\infty, -\frac{119}{2})$

The following diagram applies to Questions **5** and **6**.



5 $A' \cap B'$ is equal to:

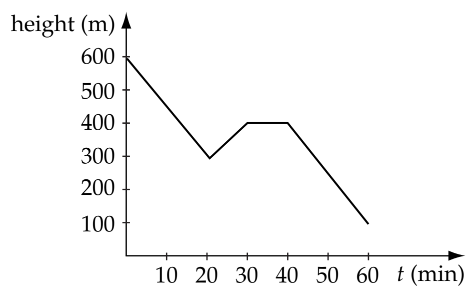
A {d, e} **B** {a, b, c, f, g, h, i}
C {a, b, c, d, e, f, g, h, i} **D** {j, k, l, m, n}
E {a, b, c, f, g, h, i, j, k, l, m, n}

6 The set {a, b, c, d, e, j, k, l, m, n} is best described as:

A $(A \cap B)'$ **B** B' **C** $A \cup B'$
D $A' \cup B'$ **E** $(A \cup B)'$

The following information relates to Questions **7** and **8**.

The rate at which the height of a hot air balloon, h (m), is changing with respect to time, t (min),is shown in the graph.



7 The rate of change in height at $t = 10$ minutes is:

A 75 m/min **B** 15 m/min **C** -15 m/min
D -30 m/min **E** 30 m/min

8 The average rate of change in height over the 60 minutes is:

A $-8\frac{1}{3}$ m/min **B** $8\frac{1}{3}$ m/min **C** 10 m/min
D -10 m/min **E** -20 m/min

9 The volume of a huge hailstone at $t = 5$ minutes is changing at the rate of $-4t$ cm³/min. The rate at which the volume of the hailstone is changing at this time is:

A 400 cm³/min **B** -20 cm³/min **C** 20 cm³/min
D -0.8 cm³/min **E** 100 cm³/min

10 If $f'(5) = 0$, $f'(x) < 0$ for $R\backslash\{5\}$ then:

A a turning point exists at $x = 5$
B a local maximum exists at $x = 5$
C a local minimum exists at $x = 5$
D a stationary point of inflection exists at $x = 5$
E $f(x)$ has an x-intercept of 5

11 The derivative of $f(x) = x^3 - \dfrac{4}{x^2}$ is:

A $f'(x) = 3x^2 + \dfrac{4}{x^3}$ **B** $f'(x) = \dfrac{8}{x^3} + 3x^2$
C $f'(x) = 3x^2 - \dfrac{4}{x^3}$ **D** $f'(x) = 3x^2 - \dfrac{8}{x^3}$
E $f'(x) = \dfrac{x^4}{4} + \dfrac{4}{x^3}$

12 $\int x^3 + 2x^2 \, dx$ is:

A $\dfrac{x^4}{4} + \dfrac{2x^3}{3} + c$ **B** $x^4 + 2x^3 + c$
C $3x^2 + 4x + c$ **D** $\dfrac{x^4}{4} - x^3 + c$
E $\dfrac{x^4}{4} + 2x^3 + c$

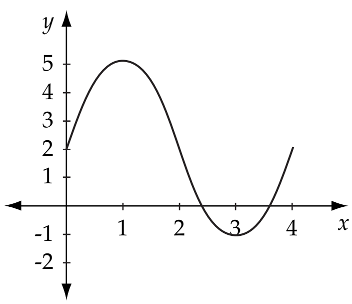
13 If ϕ is an angle between 2π and $\dfrac{5\pi}{2}$, and $\cos\phi = \sin\phi$, then ϕ equals:

A 0 **B** $\dfrac{\pi}{4}$ **C** 2π **D** $\dfrac{5\pi}{2}$ **E** $\dfrac{9\pi}{4}$

14 The value of $\cos\dfrac{5\pi}{4}$ is:

A $\dfrac{1}{\sqrt{2}}$ **B** $\dfrac{\sqrt{3}}{2}$ **C** $-\dfrac{1}{\sqrt{2}}$ **D** $-\dfrac{1}{2}$ **E** $-\dfrac{\sqrt{3}}{2}$

15 The equation of the following graph could be:



A $y = 3\sin x + 2$ **B** $y = 2\sin 3x + 2$
C $y = \sin\dfrac{\pi x}{2} + 2$ **D** $y = 3\sin\dfrac{\pi x}{2} + 2$
E $y = \sin\dfrac{3\pi x}{2}$

16 The graph of $y = 2\sin 3x + 4$ has an amplitude and period respectively of:

A 2 and $\dfrac{\pi}{2}$ **B** 2, $\dfrac{2\pi}{3}$ **C** 6, $\dfrac{2\pi}{3}$
D 6, $\dfrac{\pi}{2}$ **E** 3, π

17 The graph of $f(x) = \tan 2x$ has asymptotes at:

A $x = 0$ **B** $x = -\dfrac{\pi}{2}$ **C** $x = -\pi$
D $x = \dfrac{\pi}{4}$ **E** $x = 2\pi$

18 If $\tan x = -\dfrac{1}{\sqrt{5}}$ and $\dfrac{\pi}{2} \leqslant x \leqslant \pi$ then $\sin x$ is equal to:

A $\dfrac{1}{\sqrt{5}}$ **B** $\dfrac{1}{\sqrt{6}}$ **C** $-\dfrac{1}{\sqrt{6}}$
D $\sqrt{\dfrac{5}{6}}$ **E** $-\sqrt{\dfrac{5}{6}}$

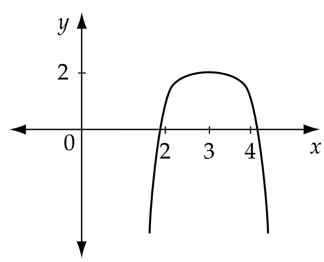
19 If the height, in metres, of water is modelled by the equation $h(t) = 3\sin\dfrac{2\pi t}{3} + 4$ then when $t = 3$, the height of the water is:

A 1 m **B** 2π m **C** 7 m
D 4 m **E** 5.5 m

20 All the solutions for $\sin x = 0$ over the domain $[0, 2\pi]$ are:

A $0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, 2\pi$ **B** $0, \pi, 2\pi$
C $\dfrac{\pi}{2}, \dfrac{3\pi}{2}$ **D** $0, 2\pi$
E $\dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}$

21 The equation of the graph shown could be:



A $y = (x-2)^2(x-4)^2$ **B** $y = (-x-3)^2 - 2$
C $y = x^2(x-2)(x-4)$ **D** $y = -(x-3)^4 + 2$
E $y = (x+3)^4 - 2$

22 The range of the function $y = |x+2| - 3$ is:

A $[-3, \infty)$ **B** $[2, \infty)$ **C** $(\infty, -3]$
D $(3, -2]$ **E** $[-2, 3]$

RETURN TO MAIN MENU

Extended answer

1 A line passes through the point (1, 3) and makes an angle of 135° with the positive direction of the x-axis.

(a) Find the equation of the line. (2 marks)

(b) Another line runs parallel to this line and passes through the point (3, 5). Find its equation. (2 marks)

(c) A third line is perpendicular to both lines and passes through the point (-1, -5). Find its equation. (2 marks)

(d) Find the points at which this line intersects the previous two lines. (3 marks)

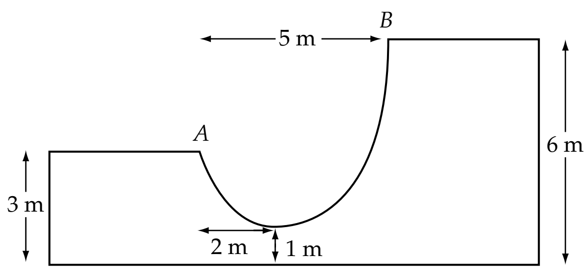
(e) Find the distance between these points of intersection. (1 mark)

2 The Shire of Safeskate has decided to build a skateboard ramp for its teenagers. The councillors are concerned about legal liability and have insisted that the shire's two young engineers, Mr Cal Culus and Ms Deri Vative, carefully inspect the ramp and prepare a safety report. The councillors believe that if the ramp has a gradient of 3 or greater at any point, it will be too dangerous to use.

The ramp is modelled by the equation

$$y = \frac{8x^2}{15} - \frac{31x}{15} + 3.$$

Below is a cross-section of the proposed ramp.



(a) Draw a graph of this function over an appropriate domain. (2 marks)

(b) Find the gradient function. (1 mark)

(c) Determine the maximum gradient possible over this domain. (2 marks)

(d) Is the ramp safe? (1 mark)

(e) The councillors have become very conservative and have decided to allow a maximum gradient of 2.5. At what height must the ramp now stop to satisfy this restriction? (2 marks)

3 A fisherman finds that the height of the tide in the harbour can be found using the equation:

$$h = 10 + 2\cos\frac{\pi t}{6}$$

where h metres is the height of the tide and t is the number of hours after midnight.

(a) What is the amplitude of this graph? (1 mark)

(b) How much time is there between consecutive high tides? (1 mark)

(c) What is the height of the tide at 6 am? (1 mark)

(d) When is the tide 12 m in height for the first time? (2 marks)

(e) What is the height of the high tide, and at what times does it occur in the first 24 hours? (2 marks)

(f) What is the difference in height between high and low tides? (1 mark)

(g) Sketch the graph of h for $0 \leqslant t \leqslant 24$. (2 marks)

(h) The fisherman knows that his boat needs a depth of 11 metres to enter the harbour. Between what hours is he able to bring his boat back into the harbour? (2 marks)

8. Exponential and logarithmic functions

Summary

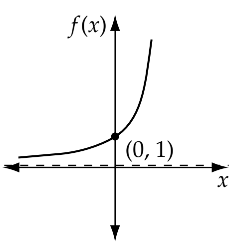
Index laws

- To multiply terms with the same base, add the powers.
  $a^m \times a^n = a^{m+n}$
- To divide terms with the same base, subtract the powers.
  $\dfrac{a^m}{a^n} = a^{m-n}$
- To raise a term to a power, multiply the powers.
  $(a^m)^n = a^{m \times n} = a^{mn}$
  $(ab)^m = a^m \times b^m = a^m b^m$
  $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
- Any number or term (except 0) when raised to the power of 0 is equal to 1.
  $a^0 = 1$ where $a \neq 0$
- When a term is moved from the numerator to the denominator or vice versa, the sign of the power changes.
  $a^{-n} = \dfrac{1}{a^n},\ a \neq 0$
- Rational exponents: $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$
- There are no values of x for which $a^x \leqslant 0$ when $a > 0$.

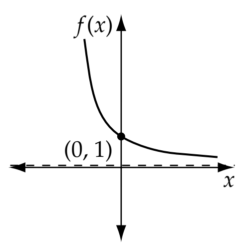
Exponential functions

- If $f: R \to R$ where $f(x) = a^x$ and $a \in R^+, x \in R$ then $f(x)$ is an exponential function.
- Graphs of exponential functions

When $a > 1$



When $0 < a < 1$

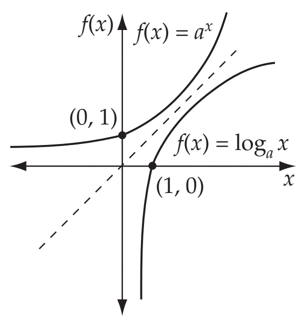


Logarithms

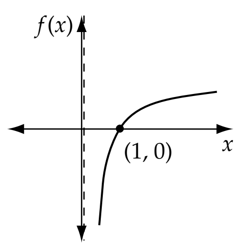
- If $y = a^x$ then $\log_a y = x$ where a = the base, x = the power or index and y = the base numeral.
- To add logarithms with the same base, multiply the base numerals.
  $\log_a m + \log_a n = \log_a (m \times n)$
- To subtract logarithms with the same base, divide the base numerals.
  $\log_a m - \log_a n = \log_a \left(\dfrac{m}{n}\right)$
- A constant in front of a logarithm can be removed by raising the base numeral to the power of that constant.
  $n \log_a m = \log_a m^n$
- When the base and base numeral are the same the logarithm is equal to 1.
  As $a^1 = a$ then $\log_a a = 1$
- $\log_a 1 = 0$
- The logarithm of a number less than or equal to zero is undefined.

Graphs of logarithmic functions

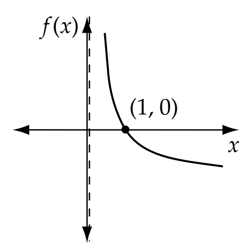
- The logarithmic function $f: R^+ \to R$ where $f(x) = \log_a x$ is the inverse function of the exponential graph with equation $f(x) = a^x$.



- When $a > 1$



When $0 < a < 1$



Frequently asked questions

How do I solve equations if different bases are involved?
It is best to use the solve function on your CAS to solve equations with different bases.

When solving an inequality, when do I reverse an inequality sign?
An inequality sign is only reversed if both sides are divided or multiplied by a negative number. If solving using CAS then it is a good idea to substitute values to check which way the inequality sign should point.

RETURN TO MAIN MENU

Study notes

- To graph an exponential or logarithmic graph, first find the asymptote and then appropriate intercepts, e.g. $y = \log_e (bx + c)$ has an asymptote at $x = -\frac{c}{b}$. Take care to note if exact intercepts are required.
- Always label any asymptotes when drawing graphs.

Cumulative Practice Examination 1

60 mins | Chapters 1–8 | Total marks: 34

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions Book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

1 The function $y = x^3$ is dilated by a factor of 3 units from the x-axis and then reflected in the y-axis.
(a) State its new equation.
(b) State the equation if it is now translated by 2 units in the positive direction parallel to the x-axis and translated by 5 units in the negative direction parallel to the y-axis.
(c) Sketch the graph for $x > 2$ and state the range.
(1 + 1 + 2 = 4 marks)

2 Solve each of the following equations.
(a) $16x^4 - 81 = 0$
(b) $2x^4 - 9x^3 - x^2 + 18x + 8 = 0$ (2 + 3 = 5 marks)

3 A box contains twenty coloured balls. Six are green, five are black, four are red, three are yellow and two are pink. Balls are drawn from the box, one at a time *without replacement*. Find the following probabilities.
(a) The first three balls drawn are all green.
(b) The first two balls drawn are the same colour.
(1 + 2 = 3 marks)

4 The following table shows the change in weight of a baby in grams, over a period of time.

time **(months)**	0	1	2	3	4	5
weight **(g)**	3500	4100	4700	5300	5900	6500

(a) Use these points to draw a graph of the change in weight over time.
(b) Find the equation of the straight line formed.
(c) What is the average rate of change in weight of the baby from birth to 5 months?
(1 + 2 + 1 = 4 marks)

5 Find the equation of the normal to the curve $y = 3x^2 + x - 3$ at the point where the tangent is parallel to the x-axis. (3 marks)

6 If $\cos x = \frac{5}{13}$, and $270° \leqslant x \leqslant 360°$, find the exact value of $\sin x$. (2 marks)

7 Simplify each of the following.
(a) $\dfrac{3^4 m^2 n^4 \times (m^3 n^2)^2}{3m^4 n^{-2}}$
(b) $\log_{10}(x + 1) + \log_{10} x^2 - 2\log_{10} x$
(2 + 2 = 4 marks)

8 Solve each of the following for x.
(a) $\log_2 64 = x$ **(b)** $\log_{10}(x + 5) = 3$
(c) $3^{2x} - 10 \times 3^x + 9 = 0$ (1 + 2 + 3 = 6 marks)

9 The graph of $y = 10^x$ is reflected in the y-axis and translated vertically -3.
(a) Find the equation of the new curve.
(b) Draw a sketch graph of the new function.
(1 + 2 = 3 marks)

Cumulative Practice Examination 2

120 mins | Chapters 1–8 | Total marks: 73

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

Multiple choice

1 The range of the function f: [-4, -1] → R where $f(x) = -2x + 3$ is:
A [-4, -1] **B** R **C** [5, 11]
D [-5, -1] **E** [5, 11)

2 Which pair of equations has the solution (1, 5)?
A $-x + y = 4$, $y = x^2 + x + 4$
B $x + 2y = 11$, $y = x^2 + 3x + 2$
C $x - y = -4$, $y = x^2 + 3x - 2$
D $x = y$, $y = x^2 + 3x - 1$
E $x + y = 6$, $y = x^2 + 2x + 2$

RETURN TO MAIN MENU

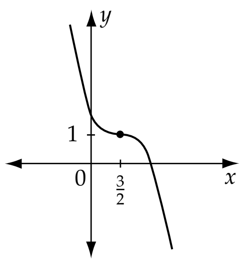
3 If $mv_1 - \frac{1}{3}mv_2 = \frac{2s}{t}$ then m is equal to:

A $\dfrac{2s}{t(v_1 - 3v_2)}$ **B** $\dfrac{6s}{(3v_1 - v_2)t}$

C $\dfrac{2s}{(3v_1 - v_2)t}$ **D** $\dfrac{6s}{(v_1 - 3v_2)t}$

E $\dfrac{\frac{2s}{t} + \frac{1}{3}v_2}{v_1}$

4 The equation that best describes the graph shown is:



A $y = (3x - 2)^3 + 1$ **B** $y = (3 - 2x)^3 + 1$

C $y = (2 - 3x)^3 + 1$ **D** $y = (x + \frac{3}{2})^3 - 1$

E $y = (3 - 2x)^3 - 1$

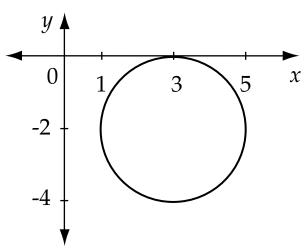
5 For the function $f(x) = \dfrac{1}{x - 4}$, select the pair of equations that represent the horizontal and vertical asymptotes of the function.

A $x = -4, y = 0$ **B** $x = -4, y = 1$

C $x = 4, y = 0$ **D** $x = 4, y = 1$

E $x = 1, y = 0$

6 The equation of the circle shown is:



A $(x + 3)^2 + (y - 2)^2 = 2$ **B** $(x - 3)^2 + (y + 2)^2 = 4$

C $(x - 3)^2 + (y + 2)^2 = 2$ **D** $(x + 3)^2 + (y - 2)^2 = 4$

E $(x - 1)^2 + (y + 5)^2 = 4$

7 Fred has a $\frac{1}{6}$ chance of being late to work on any particular day. Over the course of 84 days, the expected number of days Fred would be late is:

A 6 **B** 14 **C** 7 **D** 12 **E** 78

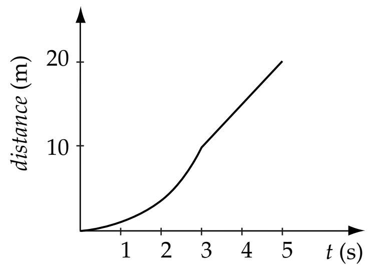
8 If we have two events P and R such that $\Pr(P) = 0.75$, $\Pr(R) = 0.25$ and $\Pr(P \cup R) = 0.95$, then $\Pr(P \cap R)$ is equal to:

A 0 **B** 0.05 **C** 1 **D** 0.1 **E** 0.95

9 A spinner is divided into five equal pieces. Two of them are coloured red, two blue, and one yellow. The spinner is spun twice. The probability of obtaining exactly one blue, if we know the first spin is not blue, is:

A $\frac{6}{25}$ **B** $\frac{9}{25}$ **C** $\frac{2}{5}$ **D** $\frac{21}{25}$ **E** $\frac{3}{5}$

10 In the graph shown the average speed from $t = 0$ to $t = 5$ is:



A 0.25 m/s **B** 0.3 m/s **C** 0.333 m/s

D 4 m/s **E** $8\frac{1}{3}$ m/s

Questions **11** and **12** refer to the following information.

The temperature of a heating coil is given by the equation $T = 20t + 20$ where T is the temperature in °C and t is the time in seconds.

11 The average rate of change in temperature with respect to time between $t = 1$ and $t = 4$ is:

A 40°C/s **B** 60°C/s **C** 25°C/s

D 20°C/s **E** 30°C/s

12 The instantaneous rate of change in temperature at $t = 4$ is:

A 20°C/s **B** 25°C/s **C** 30°C/s

D 40°C/s **E** 60°C/s

13 Differentiating $f(x) = x^3 + 3x^2 + 5$ with respect to x gives:

A $f'(x) = 3x^2 + 6x + 5$ **B** $f'(x) = 3x^2 + 6x$

C $f'(x) = \dfrac{x^4}{4} + x^3 + 5x + c$ **D** $f'(x) = 3x^2 + 6x + c$

E $f'(x) = x^2 + 3x$

14 The equation of the tangent to the curve $y = \sqrt{x} + 2x^2$ at $x = 1$ is:

A $y = \frac{1}{2}x - 1$ **B** $9x - 2y = 3$

C $y = 4.5x + 1.5$ **D** $y = 5x - 2$

E $9y + 2x = 29$

15 A farmer has 120 m of fencing wire. The biggest rectangular area that can be formed using this is:

A 30 m^2 **B** 60 m^2 **C** 90 m^2

D 900 m^2 **E** 1200 m^2

16 The value of sin $\frac{5\pi}{3}$ is:

A $-\frac{\sqrt{3}}{2}$ **B** $\frac{1}{\sqrt{2}}$ **C** $-\frac{1}{\sqrt{2}}$

D $\frac{1}{2}$ **E** $\frac{\sqrt{3}}{2}$

17 If the maximum value of a sine graph that starts at (0, 6) is 8 and it has a minimum of 4 and a period of 12, then the equation could be:

A $y = 8 \sin 12x + 6$ **B** $y = 4 \sin 6x + 3$

C $y = 2 \sin \frac{\pi x}{6} + 6$ **D** $y = 4 \sin \frac{\pi x}{12} + 6$

E $y = 8 \sin \frac{\pi x}{6} + 6$

18 Which of the following graphs will have an amplitude of 2?

A $y = \cos 3x + 2$ **B** $y = -\sin x + 2$

C $y = 2 \sin 3x - 4$ **D** $y = \sin 2x + 1$

E $y = -\cos 2x$

19 If the temperature in °C for a particular day is modelled by the equation $T = 4 \sin \frac{\pi t}{12} + 20$, where t represents the number of hours after 6 am, then which of the following statements is incorrect?

A The minimum temperature is 16°C.

B The maximum temperature is 24°C.

C The temperature has a 24 hour cycle.

D The temperature at 6 am is 20°C.

E At 2 pm the temperature is 24°C.

20 $\frac{\log_3 7}{\log_3 2}$ is closest to:

A 0.544 **B** 0.699 **C** 2.807

D 0.533 **E** 1.339

21 The equation of the graph could be:

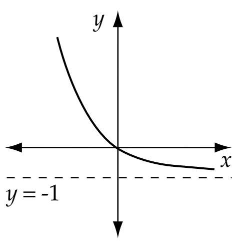
A $y = -2^x + 1$

B $y = 2^{-x} - 1$

C $y = -2^x - 1$

D $y = -2^{x+1}$

E $y = 2^{-x-1}$



22 The solution to the equation $3^x \geqslant 0.5$ is closest to:

A $x \geqslant -0.778$ **B** $x \leqslant -0.631$

C $x \geqslant 0.176$ **D** $x \leqslant -0.778$

E $x \geqslant -0.631$

23 The equation of the graph shown could be:

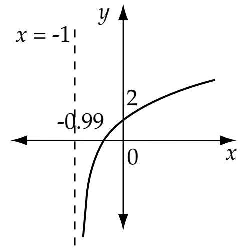
A $y = \log_{10}(x - 1) + 2$

B $y = \log_{10}(x + 0.99) + 1$

C $y = \log_{10}(x + 1) + 2$

D $y = \log_{10}(x + 1) - 2$

E $y = \log_{10}(x + 0.99) + 2$



24 $\frac{(2a^{-3}b^2)^2 \times 3^3a^{-2}}{a^5b^2}$ can be simplified to:

A $54b^2$ **B** $\frac{54b^2}{a^{10}}$ **C** $108a^4b^2$

D $\frac{108b^2}{a^{13}}$ **E** $\frac{108b^2}{a^{10}}$

25 If $2^{2x} - 4 \times 2^x = 0$ then x is equal to:

A 2 **B** 0, 2 **C** 1, -2

D 0, 2 **E** there is no solution

26 If the graph of $y = 5^x$ is reflected in the y-axis and translated right 4, the equation of the new curve would be:

A $y = 5^{-x} + 4$ **B** $y = 5^{-x+4}$ **C** $y = 5^{-x-4}$

D $y = -5^{x+4}$ **E** $y = -5^{x-4}$

27 If $16^{2x+1} = 8^{3x-1}$ then:

A $x = 1$ **B** $x = -2$ **C** $x = 7$

D $x = 2$ **E** $x = 4$

Extended answer

1 The temperature in a room is increasing since the heating is on. The temperature (T in °C) at time t (number of minutes after the heating is put on) is $T = 10a^t$.

(a) If the temperature reaches 13.4°C after 6 minutes, determine the value of a. (2 marks)

There is a thermostat on the heating system so that it cuts out when the temperature reaches 23°C.

(b) After how many minutes will the heater cut out? (2 marks)

(c) What is the temperature 10 minutes after the heating is put on? (1 mark)

Another room in the house gets a lot of sun but doesn't have the heating. The temperature in this room on a particular day follows the model

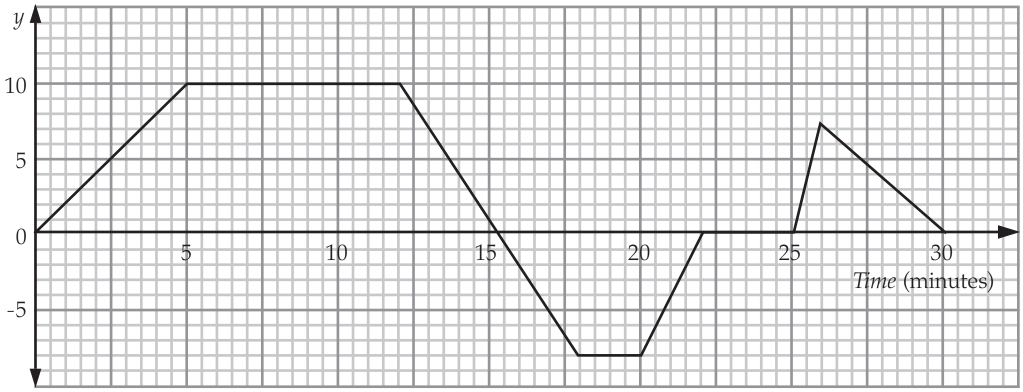
$T_2 = 8 \sin \frac{\pi t}{12} + 10$, where t is the number of hours after 8 am.

(d) What are the maximum and minimum temperatures reached in this room? (2 marks)

(e) To the nearest minute, when is the room 15°C? (2 marks)

(f) For how many minutes in the first 12 hours is the room warmer than 15°C? (2 marks)

2 Johnny, the household rodent, rushes around the home collecting food scraps. He needs to be careful to avoid traps and detection by the homeowners. As a result he often stops and starts, and he also changes direction constantly. The graph below shows his typical progress.



Time is measured in minutes and positive displacement is defined as the position to the right of the starting point.

Assuming that the given graph is a displacement–time graph, find the following.

(a) the times at which Johnny was stationary (3 marks)

(b) the times at which Johnny changes direction (2 marks)

Now assume the graph is a velocity–time graph.

(c) Find the times at which Johnny was stationary. (2 marks)

(d) Find the times at which Johnny changes direction. (2 marks)

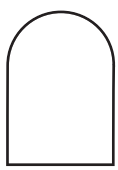
Now go back to thinking that the graph is a displacement–time graph and find:

(e) Johnny's displacement at the end of 30 minutes (1 mark)

(f) the total distance travelled during the 30 minutes (2 marks)

(g) Use the displacement–time graph and the fact that the velocity is the rate of change of displacement with respect to time to draw a velocity–time graph of Johnny's journey. (2 marks)

3 A window manufacturer designs window panes to the plan at right. The window pane consists of a rectangle and a semicircle. The manufacturer wishes to build the window so that it allows in the maximum amount of light. Let x cm be the radius of the semicircle and y cm be the height of the rectangular part of the window pane.



(a) Find an expression for y in terms of x given that the perimeter of the pane is 50 cm. (2 marks)

(b) Show that the area of the window is given by:

$$A(x) = \frac{100x - 4x^2 - \pi x^2}{2}$$ (2 marks)

(c) Find the dimensions of the window so that the area is a maximum. (3 marks)

(d) The perimeter of the window pane can vary according to the job specifications. Show that for any perimeter P the maximum area of the window will be:

$$A = \frac{P^2}{2(\pi + 4)}$$ (3 marks)

4 In this question you are going to simulate the tossing of a coin by using your CAS to produce random numbers of 0 and 1 only. Assign 0 to Heads.

(a) Write the expression you will use to produce these numbers. (1 mark)

(b) You are going to simulate tossing the coin 100 times. Estimate:

- **(i)** the average run length of the same result (1 mark)
- **(ii)** the longest run of the same result that will occur. (1 mark)

(c) Now conduct the simulation and record your results. (1 mark)

(d) Calculate the average run length. (1 mark)

(e) What was the longest run obtained? (1 mark)

(f) If you conducted the experiment again, how do you think your results would compare to the first one? (1 mark)

(g) Theoretically, what is the probability of getting:

- **(i)** two heads in a row
- **(ii)** three heads in a row
- **(iii)** four heads in a row
- **(iv)** five heads in a row (2 marks)

(h) Theoretically, what is the probability of getting, as the first three results:

(i) HTH **(ii)** THT **(iii)** TTH

(2 marks)

9. Probability applications

Summary

Counting techniques

- Two events are mutually exclusive if membership of one event excludes membership of the other.
- The addition principle tells us that if two events are mutually exclusive then the total number of ways those two events can occur is the sum of the individual number of ways each of the events can occur.
- The multiplication principle tells us that if one event can occur in m ways and this is followed by another event that can occur in n ways then the two-stage event can occur in $m \times n$ different ways. This is also referred to as the fundamental principle of counting.
- $n!$ (n factorial) $= n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1 =$ number of ways n objects can be arranged in a row.
- $0! = 1$

Permutations

- With permutations (arrangements) order is important, e.g. ABC is regarded as different from CBA.
- The number of permutations of n objects taken r at a time can be represented by ${}^nP_r$.
- ${}^nP_r = \dfrac{n!}{(n-r)!}$
- The number of different ways of arranging n objects when we have n_1 of type 1, n_2 of type 2, …, n_r of type r is $\dfrac{n!}{n_1!n_2!\ldots n_r!}$ where $n_1 + n_2 + \ldots n_r = n$
- When n different objects are arranged in a circle this can be done in $\dfrac{n!}{n} = (n-1)!$ distinct ways.

Combinations

- With combinations (selections) order is not important, e.g. ABC is regarded as the same as CBA.
- ${}^nC_r = \binom{n}{r}$ = the number of selections of n objects taken r at a time.
- ${}^nC_r = \dfrac{{}^nP_r}{r!} = \dfrac{n!}{(n-r)!r!}$
- ${}^nC_r = {}^nC_{n-r}$
- The nth row of Pascal's triangle gives us the number of ways $n-1$ objects can be selected 0, 1, 2, …, $(n-1)$ at a time.
- The ideas from permutations and combinations can be applied to probability questions using the basic probability definition

  $$\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Applications of conditional probability

- The law of total probability is another way of expressing the probability of event B in a two-event experiment. It can be expressed as follows.

  $\Pr(B) = \Pr(B \mid A)\Pr(A) + \Pr(B \mid A')\Pr(A')$
- A stochastic process is a mathematical model that evolves over time, based on probabilities associated with the event.
- A Markov chain is a special stochastic process in which the next state of the system depends only on the current state.
- Transition matrices can be used to model Markov chains and simplify the calculations involved with them.

Frequently asked questions

How can I tell when to use a permutation and when to use a combination?

The easiest way is to ask yourself 'Does order matter?' If the answer is yes, then you need to consider the question from a permutations point of view. If the answer is no, then you should use a combinations approach.

What do I do if the probability changes for each stage of an experiment?

In cases like this you are probably best off to draw up a tree diagram, placing the appropriate probabilites on each branch. Then go through and identify the branches that result in the desired outcome and add together these probabilities.

Study notes

- Read the questions carefully, looking for key words that will help you decide whether the permutation or combination formula can or should be used.

RETURN TO MAIN MENU

Cumulative Practice Examination 1

60 mins | Chapters 1–9 | Total marks: 34

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions Book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

1 Determine the equation for each of the following.

(a) $f(x) = \frac{1}{x}$ is dilated by a factor of 3 and translated 3 units to the right and reflected in the x-axis.

(b) A quartic polynomial has a point of inflection at (3, 0) and passes through (0, 5) and (2, 0).

(c) The function $f(x) = \frac{1}{x^2}$ is reflected in the x-axis, translated 3 units to the left and 2 units up.

(d) A quartic function has turning points at (3, 0) and (-2, 0) and a y-intercept of 4.

(1 + 2 + 1 + 2 = 6 marks)

2 (a) Sketch the graphs of $3x - y = 6$ and $y = 2x - 5$ on the same set of axes.

(b) Algebraically find the point of intersection between these two lines. (2 + 2 = 4 marks)

3 The rate of change in temperature of an object (°C) can be expressed as the function $T(x) = 3x^3 + 2x + 20$ where x is in minutes. Find:

(a) the initial temperature of the object

(b) the temperature after 3 minutes

(c) the average rate of change in the temperature of the object in the first 3 minutes.

(1 + 1 + 2 = 4 marks)

4 Write down the exact values of each of the following.

(a) $\sin \frac{5\pi}{6}$ **(b)** $\tan \frac{3\pi}{4}$

(c) $\cos \frac{\pi}{2}$ **(d)** $\sin \frac{7\pi}{3}$

(1 + 1 + 1 + 1 = 4 marks)

5 A spherical balloon is being inflated. Find the rate of change of the volume with respect to the radius when:

(a) the radius is 6 cm

(b) the volume is 36π cm^3 (2 + 2 = 4 marks)

6 The local darts club is sending a men's pair and a women's pair to the country championships. In each pair the first player chosen will be the lead player, who always plays first out of the pair. There are twelve men and six women competing for positions on the teams.

(a) In how many ways can the men's team be chosen?

(b) In how many ways can the women's team be chosen?

(c) How many different pairs can be chosen in total?

(d) How many different teams can the club send to the championships? (1 + 1 + 1 + 1 = 4 marks)

7 Consider the word MEGALOMANIAC. Write an expression that could be used to calculate each of the following situations where each letter is used.

(a) The number of different ways the letters can be arranged if there are no restrictions.

(b) The number of different ways the letters can be arranged if the Ms are together.

(c) The numbers of arrangements in which the Ms are together and the As are together.

(1 + 1 + 1 = 3 marks)

8 The Student Council consists of a four-person executive and six other members. Write expressions that could be used to calculate the number of ways the group could sit around a circular table if:

(a) there were no restrictions

(b) the executive had to sit together.

(1 + 1 = 2 marks)

9 The newest automobile number plates in New South Wales consist of three letters followed by two numbers and then another letter. An example is APZ 34W. Write a simplified expression that could be used to calculate the probability that such a number plate:

(a) contains all different letters

(b) contains different digits

(c) begins and ends with A (1 + 1 + 1 = 3 marks)

RETURN TO MAIN MENU

Cumulative Practice Examination 2

120 mins Chapters 1–9 Total marks: 68

Fully worked solutions to *all* of these questions are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

Multiple choice

1 The equation of the line passing through the points (-2, 5) and (4, 7) is:

A $y = \frac{1}{3}x + 5$ **B** $3y - x - 17 = 0$
C $3y - x = 13$ **D** $y = 3x + 11$
E $y = 9x + 23$

2 The graph of the function $y = 3(x - a)(x + 2)$ has a y-intercept of:

A 6 **B** -6 **C** $-2a$ **D** $-6a$ **E** $6a$

3 The graph of $y = 2(5 - 2x)^3 + 4$ has a point of inflection at:

A (2, 5) **B** (5, 4) **C** $(-\frac{5}{2}, -4)$
D $(\frac{5}{2}, 4)$ **E** (2, 4)

4 The domain and range of $f: [1, 7) \to R$ where $f(x) = 2(x - 4)^3 - 3$ are:

A [1, 7), [-57, 51] **B** R, R
C [1, 7), (-57, 51] **D** (1, -57), (7, 51)
E [1, 7), [-57, 51)

5 The equation of the circle with centre at (1, -2) and radius of 3 units is:

A $(x + 1)^2 + (y - 2)^2 = 3$ **B** $(x - 1)^2 + (y + 2)^2 = 3$
C $(x - 1)^2 + (y + 2)^2 = 9$ **D** $(x + 1)^2 + (y + 2)^2 = 9$
E $(x - 1)^2 + (y - 2)^2 = 9$

6 The equation of a hyperbola with asymptotes at $x = 2$ and $y = -1$ could be:

A $y = \dfrac{2}{x - 2} - 1$ **B** $y = \dfrac{-1}{x + 2} - 1$

C $y = \dfrac{2}{x - 1}$ **D** $y = \dfrac{1}{x + 1} - 2$

E $y = \dfrac{-1}{x + 2} + 1$

7 A tally is kept of the make of cars passing the front gate of the school. The results for one period of observation were:

Make	Holden	Ford	Mazda	Mitsubishi	Toyota
Number recorded	33	27	11	18	31

Based on these results, the probability that the next car will be a Ford or a Mitsubishi is:

A $\frac{2}{5}$ **B** $\frac{9}{40}$ **C** $\frac{3}{20}$ **D** $\frac{1}{5}$ **E** $\frac{3}{8}$

8 An egg packing company knows from past experience that one-sixth of all eggs pass through the inspection point with minor flaws. A simulation was run where H1 and T1 represented flawed eggs. Sixty trials were conducted with the following results.

H4	H1	T1	H4	H6	H3	T2	T1	H1	T3	T2	H4
T3	T4	H4	T1	H5	T2	T6	H2	T2	H3	H6	T3
T1	H1	H3	T2	H3	T1	H5	T4	T6	H1	H1	H6
T4	T4	H6	T5	H6	T4	T4	H1	H6	T2	T4	H3
T2	H6	T3	T5	H6	T2	H3	T4	T6	H6	T1	H5

Each row represents a dozen eggs.
Based on these results, the probability that a dozen eggs contains more than two eggs with minor flaws is:

A $\frac{2}{5}$ **B** $\frac{1}{5}$ **C** $\frac{1}{6}$ **D** $\frac{1}{3}$ **E** $\frac{4}{5}$

9 A ten-sided die labelled 1–10 is rolled once. The probability that the first number rolled is a composite number is:

A $\frac{5}{12}$ **B** $\frac{1}{2}$ **C** $\frac{7}{12}$ **D** $\frac{1}{6}$ **E** $\frac{1}{5}$

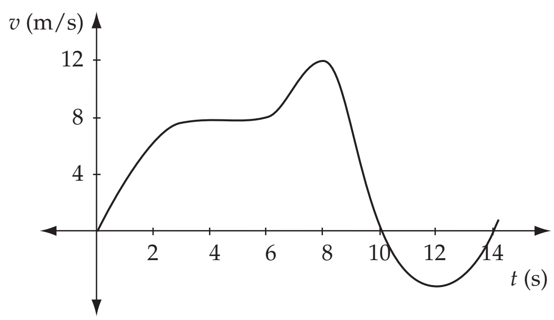
10 Which of the following statements does not represent change in volume with respect to time?

A 4 cm^2/min **B** 2 L/hour
C -200 cm^3/min **D** 1000 L/s
E 50 kL/day

11 The average rate of change of the function $y = x^3 + 2x + 5$ between $x = 3$ and $x = 6$ is:

A 195 **B** 233 **C** 38 **D** 32.5 **E** 65

12 The times at which the following body changes direction are:



A $t = 3$, 8 and 12 seconds
B $t = 8$ and 12 seconds
C $t = 10$ and 14 seconds
D $t = 3$, 8, 10 and 12 seconds
E $t = 8$, 10 and 12 seconds

13 $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ for $f(x) = 3x^2 + 5$ when $x = 1$ gives:

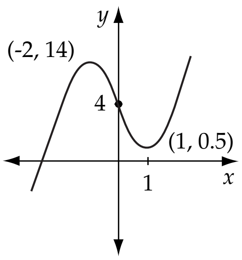
A 0 **B** 8 **C** 6
D 1 **E** limit undefined

14 The coordinates of the point(s) at which the tangent to the curve $y = 3x^3 - 4x$ is/are parallel to the line $y = 2x + 4$ is/are:

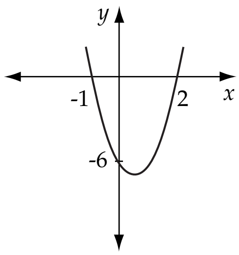
A (-0.816, 1.634), (0.816, -1.634)
B (1.35, 2)
C (0.667, -1.778)
D (-0.667, 1.778)
E (0.471, 1.571), (-0.471, 1.571)

15 If $f(x) = x^3 + \frac{3}{2}x^2 - 6x + 4$, the graph of $f'(x)$ would be:

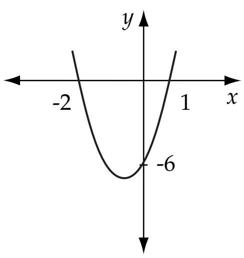
A



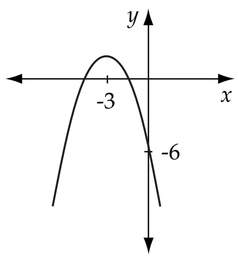
B



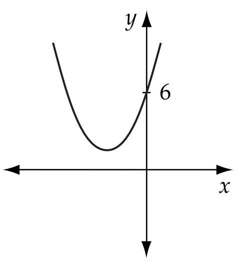
C



D



E



16 The graph of $y = 4 \cos x + 7$ has a range of:

A [4, 7] **B** [-3, 11] **C** [3, 7]
D [3, 11] **E** [0, 7]

17 The graph of $f(x) = \tan \frac{\pi x}{3}$ has an x-intercept at:

A $(\frac{\pi}{3}, 0)$ **B** $(3, 0)$ **C** $(\frac{3}{2}, 0)$
D $(\frac{3\pi}{2}, 0)$ **E** $(-\frac{5}{2}, 0)$

18 The sum of the solutions for $\cos x = \frac{\sqrt{2}}{2}$ over the domain $(0, 2\pi)$ is:

A $\sqrt{2}$ **B** 2π **C** π **D** $\frac{3\pi}{2}$ **E** $\frac{\pi}{2}$

19 The graph of $y = \log_{10} x$ is reflected in the x-axis and translated vertically by 2. The equation of the asymptote of the new curve would be:

A $y = 2$ **B** $y = -2$ **C** $x = 2$
D $x = -2$ **E** $x = 0$

20 $27^{3x} \times 9^{2x+1}$ can be expressed as:

A 3^{5x+5} **B** 3^{13x+2} **C** 3^{11x+1}
D 3^{7x+4} **E** 3^{13x+1}

21 $\frac{36^{m+2} \times 4^m}{3^{m+2}}$ can be simplified to:

A 12^{m+2} **B** $3^{m+2}4^{2m+2}$ **C** $9^{m+2}4^m$
D $12 - 4^m$ **E** $3^{2m+4}4^{2m+2}$

22 $^{11}P_5$ is equal to:

A $\frac{11!}{5!}$ **B** $\frac{11!}{6!}$ **C** $11! - 5!$
D $\frac{1!}{5!}$ **E** $\frac{1!}{6!}$

23 Five boys and four girls go to the cinema. In how many ways can they sit in a row if boys and girls are to alternate?

A $9!$ **B** $\frac{9!}{5! \times 4!}$ **C** $5! + 4!$
D $5! \times 4!$ **E** $9! - (5! \times 4!)$

24 The value of the unknown in $^{10}C_3 = \frac{^{10}P_3}{t}$ is:

A 3! **B** 10! – 7! **C** 7!
D 10! – 3! **E** 3

25 A box contains five blue balls, four yellow balls and three white balls. Three balls are drawn without replacement from the box. The probability that there will be one of each colour is:

A $\frac{^5P_1 \times {^4P_1} \times {^3P_1}}{^{12}P_3}$ **B** $\frac{^5C_1 + {^4C_1} + {^3C_1}}{^{12}C_3}$

C $\frac{3}{13}(^5C_1 + {^4C_1} + {^3C_1})$ **D** $\frac{3}{13}(^5C_1 \times {^4C_1} \times {^3C_1})$

E $\frac{^5C_1 \times {^4C_1} \times {^3C_1}}{^{12}C_3}$

26 The probability that a three-letter word formed by taking letters, without replacement, from the word SENSATIONAL, ends in a vowel is:

A $\frac{10 \times 9 \times 8}{11!}$ **B** $\frac{11!}{8! \times 3!}$

C $\frac{11!}{8! \times 3! \times (2!)^3}$ **D** $\frac{10 \times 9 \times 5 \times (2^3)!}{11!}$

E $\frac{8! \times 3! \times (2!)^3}{11!}$

27 A family consisting of a mother, a father, two daughters and four sons is seated at a circular table. The probability of the sons sitting together is:

A $\frac{4}{35}$ **B** $\frac{1}{14}$ **C** $\frac{1}{70}$ **D** $\frac{1}{2}$ **E** $\frac{1}{1680}$

Extended answer

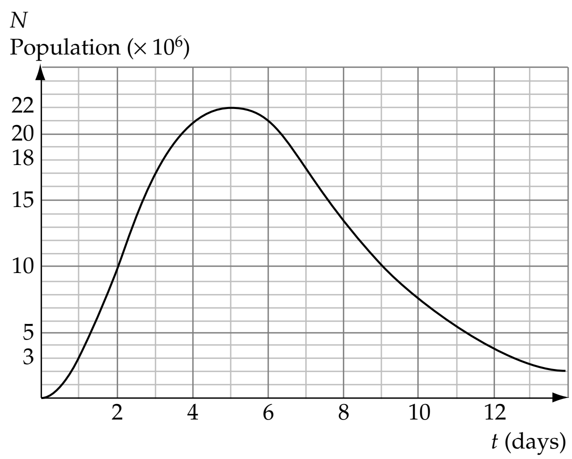
1 **(a)** Rearrange the following equation into the standard form for a circle:
$x^2 + 4x + y^2 - 4y = 8$ (3 marks)

(b) The circle is translated 3 units up and 2 units to the left. Determine the new equation. (1 mark)

(c) Determine the domain and range of the first quadrant of the transformed circle. (2 marks)

(d) Determine the equation of the inverse function of this section of the graph and state its domain and range. (3 marks)

2 The population N of a bacterial species responsible for infections of the throat before and during antibiotic treatment is illustrated below.



(a) Use the graph to estimate the bacterial population at $t = 2$ days. (1 mark)

(b) Calculate the average rate of change of the bacterial population from $t = 1$ to $t = 3$ days. (2 marks)

(c) Calculate the rate at which the bacterial population is changing at $t = 3$ days. (2 marks)

(d) At what time is the rate of change of the bacterial population equal to zero? (1 mark)

(e) At what time is the population decreasing most rapidly? Find this rate. (2 marks)

(f) Sketch the graph of the rate of change of the bacterial population against time (days). (2 marks)

3 The number of bacteria in a culture is given by $N = N_0 2^{kt}$ where k is a constant and t is the number of days the culture has been left in a particular place. There are 100 bacteria present at the start of the experiment and in one day this has grown to 400.

(a) Determine the values of N_0 and k. (2 marks)

(b) How many bacteria will there be in 3 days? (1 mark)

(c) When will the number of bacteria reach 10 000? (2 marks)

Another culture is placed in a more sterile environment and the number of bacteria is represented by $B = B_0 2^t$, with t again representing the number of days the culture has been left. This culture starts the experiment with 200 bacteria present.

(d) When will the population of this culture have doubled in size? (2 marks)

(e) Will the two cultures ever have the same number of bacteria at the same time, and, if so, when and how many? (2 marks)

4 The executive of the hockey club consists of president, vice president, secretary, treasurer and two other members. The executive is to be selected from the twenty-eight adult members of the club, thirteen of whom are female.

Find the exact probability that the executive:

(a) is made up of three males and three females (1 mark)

(b) contains more females than males (2 marks)

(c) is all female. (1 mark)

Karen, a female, is the first person elected, taking the position of president. Find the exact probability now that the executive:

(d) is made up of three males and three females (1 mark)

(e) contains more females than males (1 mark)

(f) is all female. (1 mark)

Wendy, a female, is next elected to the position of vice president. Find the exact probability now that the executive:

(g) is made up of three males and three females (1 mark)

(h) contains more females than males (1 mark)

(i) is all female. (1 mark)

Lauren, a female, takes on the third position—secretary. Find the exact probability now that the executive:

(j) is made up of three males and three females (1 mark)

(k) contains more females than males (1 mark)

(l) is all female. (1 mark)